Modelling a wool scour bowl

J. F. Caunce* S. I. Barry* G. N. Mercer*
T. R. Marchant†

(Received 20 October 2005; revised 13 June 2006)

Abstract

Wool scouring is the process of washing dirty wool after shearing. Our model simulates, using the advection-diffusion equation, the movement of contaminants within a scour bowl. The effects of varying the important parameters are investigated. Interesting, but simple, relationships are found which give insight into the dynamics of a scour bowl.

Contents

*School of Physical, Environmental and Mathematical Sciences, University of New South Wales at ADFA, Canberra, Australia. mailto:s.barry@adfa.edu.au

†School of Mathematics and Applied Statistics, University of Wollongong, Wollongong, Australia.

See http://anziamj.austms.org.au/V47EMAC2005/Caunce3 for this article, © Austral. Mathematical Soc. 2006. Published June 22, 2006. ISSN 1446-8735

1 Introduction

Wool scouring is the process of washing shorn wool to remove unwanted contaminants. These contaminants fall into three categories: suint, grease and dirt. Suint is water soluble and removed in the first bowl. Grease is removed with the aid of detergents. Insoluble dirt (dust, stones and vegetable matter) is removed from the wool through agitation.

The conventional wool scouring machine usually consists of two to four scour bowls and three rinse bowls. Figure 1 shows a schematic of a scouring machine with three scour and three rinse bowls. Wool enters at the feed hopper at the left of the figure, travels through the scour and rinse bowls and leaves through the dryer. Water is transported from bowls $6 \to 5 \to \cdots \to 1$ for efficiency. However, within each bowl, water flows in the same direction as the wool, hence the flow is not a classic 'separation' process. For example, in Figure 1 the water in bowl 3 proceeds from left to right within the bowl before being pumped to the left of bowl 2.

Scour bowls are filled with a liquor primarily consisting of water, but also soaps, detergents and the contaminants that have been removed from the wool. The wool is driven as a saturated mat along the top of the bowl, by sets of harrows. Both the wool and water enter from the same side of the bowl. The agitation frees the dirt which settles down the bowl to a set of three to four settling tanks as shown in Figure 2. Periodically each tank

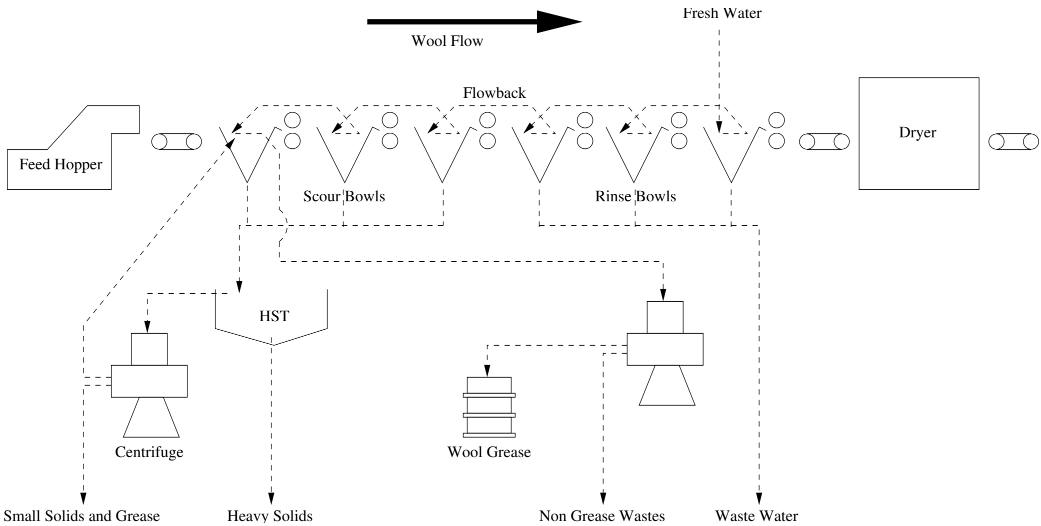

Figure 1: The conventional scouring system

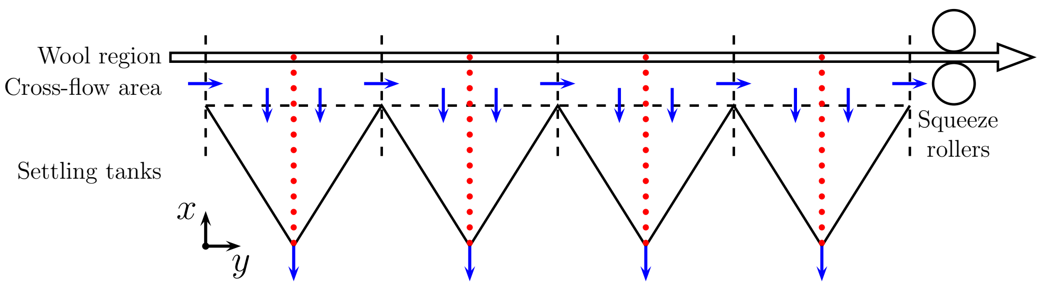

Figure 2: The conventional scour bowl. With dots showing the discretisation points of the model and arrows showing the liquor flow.

is drained to purge the collecting contaminants. The liquor is cleaned and recycled to the bowls. The purges occur according to a drainage pattern, that specifies a purge cycle for each tank, shown in Figure 3, where the time of purges are indicated by the peaks.

Grease is swollen by the detergents in the bowls, but most of the grease is removed at the squeeze rollers by the rush of water through the wool caused by the rollers. The grease rich liquor at the top of the bowls and the squeeze rollers is sent to a centrifuge where the grease is separated to be sold as lanolin.

Previous work has modelled the scouring system to find the average conditions of the whole system at steady state. These models treat each scour bowl or even the whole scouring machine as a single entity [3, 4]. The limitations of these models are that: they cannot find the state of the system at any time before it reaches steady state or explore the effect of the periodic drainage; and it is not possible to see how the contaminants move within each scour bowl. A study group report, [1], briefly considered a time dependent model of a scour bowl compartmentalised into eight regions, but a more complete model is needed.

The aim of our model is to evaluate the flow of contaminants accurately enough to reflect the important features of the system, but be efficient enough to allow a computer simulation to run quickly. The scour bowl is not an exact system, many of the flows and parameters associated with the scour bowl are very variable, with some of the flows turbulent. Thus, fully three dimensional fully turbulent flow models, for example, would provide few useful results compared to simple models whilst taking far longer to compute. We believe that the simpler model developed in this paper allows a better understanding how contamination in wool is effected by operating parameters. For example, Figures 5–7 in the results section, show physical output of contamination in the wool versus the key operating parameters of wool and settling velocities.

Our model looks at only one scour bowl, rather than the entire scour

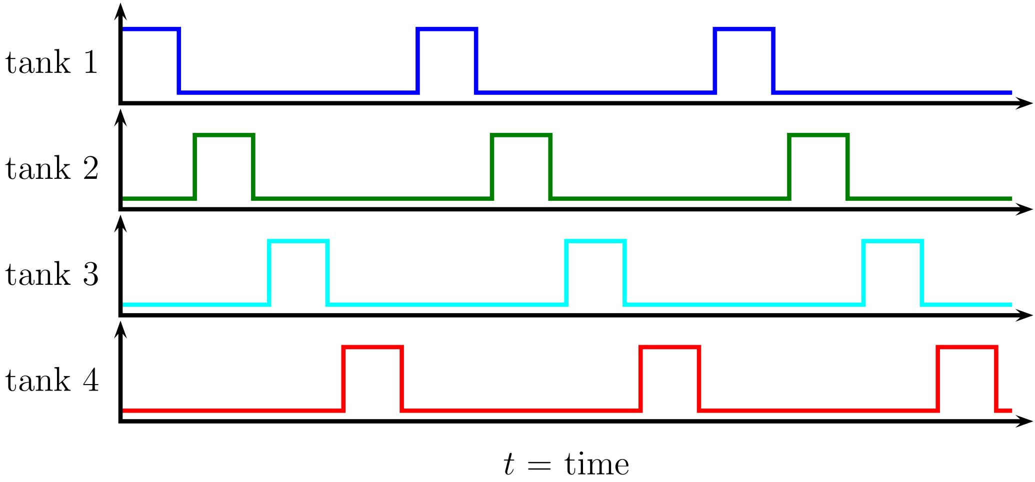

Figure 3: The purge cycles; the rate of drainage from each tank against time.

machine as in [3, 4]. Each bowl consists of four tanks separated into wool, cross-flow area and settling tank regions (Figure 2). The model allows the variation of many of the important parameters, such as; wool velocity w_{wool}; the initial contamination concentration of the wool c^w_{in}; the input flow rate of liquor into the bowl q_{in}; the concentration of input liquor c_{in}; the particle settling velocity v_{set}; and the drainage pattern, which specifies the purge cycles.

This paper discusses first the mathematical model, then the numerical discretisation method, followed by results and conclusions.

2 Discretisation model

The concentration of contaminants $c(x, y, t)$ throughout the bowl with time t is modelled by the advection-diffusion equation

$$\frac{\partial c}{\partial t} = D\left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2}\right) - \frac{\partial}{\partial x}(vc) - \frac{\partial}{\partial y}(wc)\,, \tag{1}$$

where v is vertical velocity and w is the horizontal velocity of the contaminants and D is a small constant diffusion term that reflects not only pure diffusion, but also some realistic mixing within the fluid due to the harrows.

Numerical finite differences are used to discretise the scour bowl into finite points, shown in Figure 2. Each point represents the concentration of contamination in a region around it for the width of the tank. The x discretisation width Δx is decided by balancing the conflicting aspects of accuracy and calculation time. As there is one column of points in each tank, the y discretisation width Δy is rather crude. It is assumed the diffusion term $D\frac{\partial^2 c}{\partial y^2}$ in equation (1) is negligible, given the length scales Δy, as the effect of horizontal advection $w\frac{\partial c}{\partial y}$ is far greater than diffusion. For simplicity,

$$c^k_{j,i} = c(j\Delta x, i\Delta y, k\Delta t) \tag{2}$$

is used to denote the contamination at each discretised point, and similarly for other variables.

Before the advection-diffusion equation is solved, the velocities v and w need to be found for all times. The vertical velocity v takes into account both the flow rate of the liquor and the settling velocity of the contamination v_{set}. The vertical velocity at a particular height is calculated by dividing the drainage flow rate d^k_i by the cross-sectional area A_j of the tank at that height, to find the flow's velocity, and adding the settling velocity:

$$v^k_{j,i} = \frac{d^k_i}{A_j} + v_{\text{set}}\,. \tag{3}$$

The horizontal flow rates through the cross flow area, $q_{i,i+1}$ from tank i to $i+1$, must be calculated given a known input flow rate q_{in} and the known rates of drainage d_i^k. The flow rates are found by the simultaneous solving of a set of conservation of mass equations, one for each tank. For a bowl of four tanks the four equations are

$$q_{\text{in}} - q_{12} - d_1 = 0\,, \qquad q_{12} - q_{23} - d_2 = 0\,, \tag{4}$$

$$q_{23} - q_{34} - d_3 = 0\,, \qquad q_{34} - q_{\text{out}} - d_4 = 0\,, \tag{5}$$

where q_{out} is the unknown output flow rate from tank 4. Once the flow rates are known the horizontal velocities are found by dividing by the horizontal cross-sectional area of the cross-flow area B

$$w_{j,i}^k = \frac{q_{i,i+1}}{B}\,. \tag{6}$$

To solve this system satisfactorily, the bowl is divided into three regions, the wool region, cross-flow area and the settling tanks. The regions in each tank are solved with a slightly different numerical method.

In the settling tank region there is no flow in the y direction, so, for a rectangular settling tank, the advection-diffusion equation simplifies to

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2} - \frac{\partial}{\partial x}(vc)\,. \tag{7}$$

However, in a more realistic pyramid shaped tank, the concentration of contamination increases as the the cross-sectional area decreases down the tank. An area-weighted advection-diffusion equation, which takes into account the shape of the tank is

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2} + \frac{A'}{A}D\frac{\partial c}{\partial x} - \frac{A'}{A}vc - v\frac{\partial c}{\partial x}\,, \tag{8}$$

where A is the cross-sectional area of the tank at height x and A' is its derivative.

The boundary condition at the top of the region is simply the continuous contamination flow down from the cross-flow area. The boundary condition at the bottom depends on the drainage regime in the tank. At times when there is no drainage occurring from the tank, the no flux boundary condition is

$$D\frac{\partial c}{\partial x} - vc = 0\,. \tag{9}$$

At other times, when drainage does occur, the boundary condition $\frac{\partial c}{\partial x} = 0$ allows contamination to advect unhindered through the bottom.

Tests were run on the accuracy of second to fifth order finite difference schemes for the settling tanks, comparing results with simple exact solutions. A fourth order scheme was chosen because it gives a satisfactory degree of accuracy at the bottom boundary and is not too computationally intensive.

The cross-flow area is the region of the scour bowl between the wool and the settling bowls. In this region the scour liquor flows both horizontally from tank to tank and vertically down from the wool and into the settling tanks as they drain, so the full two dimensional advection-diffusion equation is used, as in equation (1). The equation is easiest to solve in this region because it has no complicated boundary conditions, so it can be numerically approximated with first order finite difference methods for time and y-space and a second order method for x-space. The boundary conditions on the top and bottom are obtained by continuity with the settling tank region and the wool region. The boundary condition at $y = 0$ is

$$c(x, 0, t) = c_{\text{in}} \tag{10}$$

where c_{in} is the concentration of contamination of the input flow into the cross-flow area. As there is only a first order derivative in y, $\partial c/\partial y$, the first order finite difference method used in y-space means no boundary condition is required at the end of the bowl.

Unlike the rest of the bowl which is discretised into points Δx apart, the wool is represented by a single point in each tank. This is reasonable because the wool region is well mixed by the action of the harrows. This also makes the no-flux boundary condition on top of the wool easier to apply; with the same discretisation as the rest of the bowl, this region would have required a more accurate and computationally intensive method than has been used. The governing equation for the wool region is a simple mass balance of incoming and outgoing contamination,

$$\frac{\partial c}{\partial t} = -\frac{F(0,y)}{H} - \frac{w_{\text{wool}}}{\Delta y}\left(c(x,i\Delta y) - c(x,0)\right), \tag{11}$$

where H is the height of the wool, and

$$F(x,y) = D\frac{\partial c}{\partial x} - v(x,y)c(x,y)$$

represents the flux of contamination between the wool and the scour liquor. In the finite difference scheme, equation (11) becomes

$$c_{n,i}^{k+1} = \frac{w_{\text{wool}}\Delta t}{\Delta y} c_{n,i-1}^{k} + \left(1 - \frac{w_{\text{wool}}\Delta t}{\Delta y}\right) c_{n,i}^{k} - \frac{\Delta t}{H} F_{n,i}^{k}. \tag{12}$$

The only boundary condition is at the beginning of the wool to set the contamination of the incoming wool.

Various values of $\Delta x, \Delta t$ are used to generate results, with these terms modified to maximise convergence and minimise iteration time depending on the parameter values.

3 Results

Figure 4 illustrates the typical results the model generates, showing concentration of contamination c in the first tank versus time t. These results are, as indicated by the schematic beside the graph, of the concentration at the bottom of a rectangular shaped tank. The three lines represent simulations

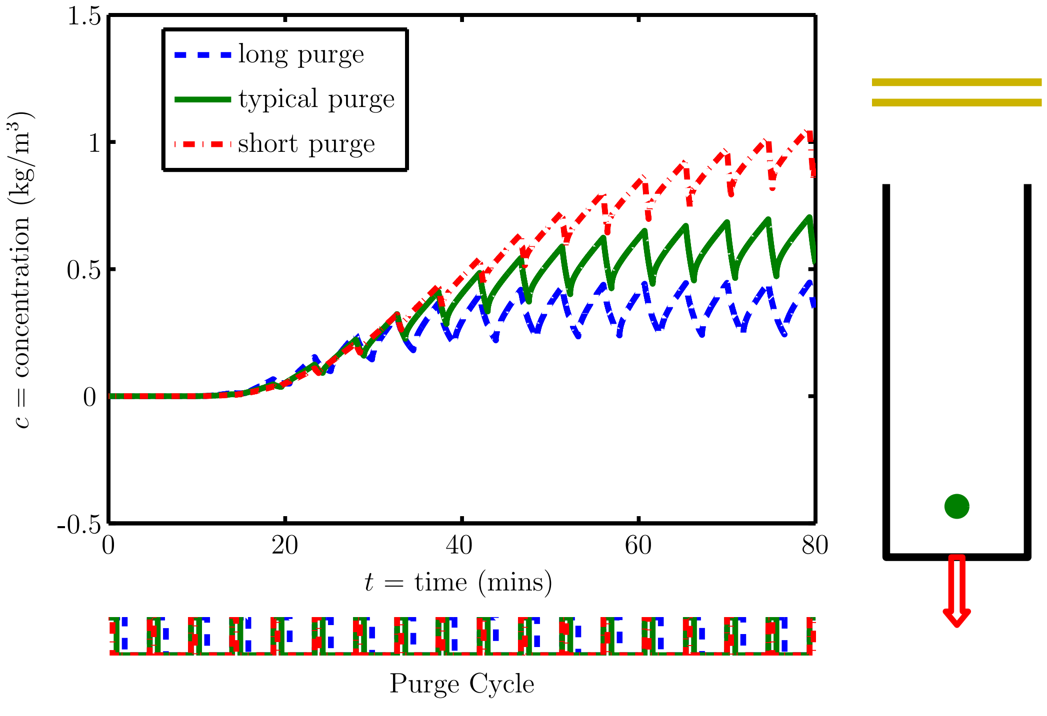

Figure 4: The build up of contamination at the bottom of the first tank with three different purge cycles.

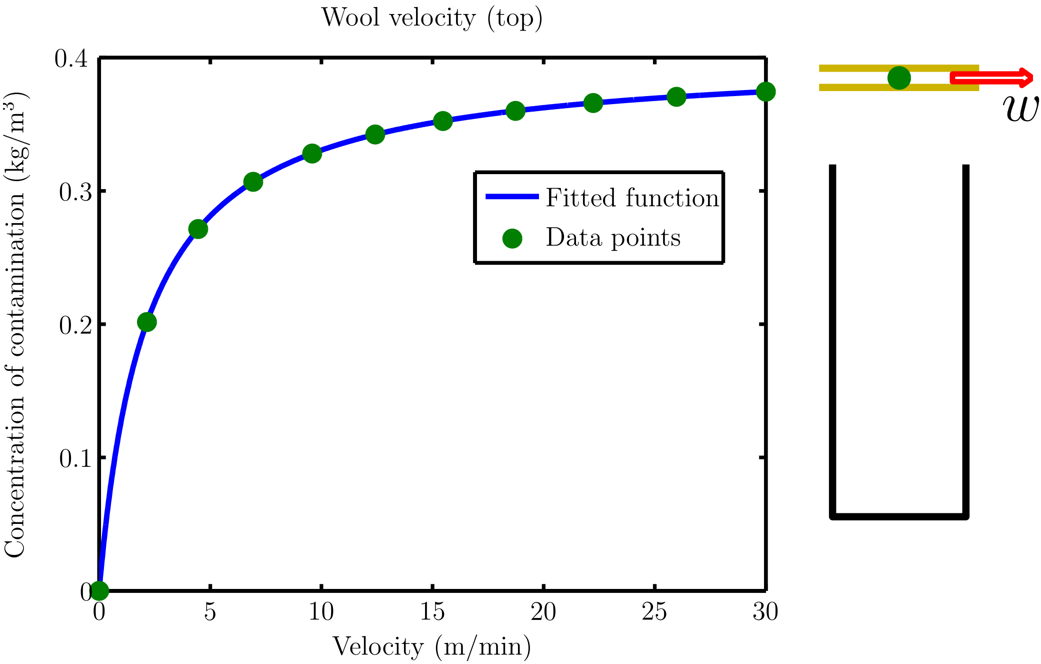

Figure 5: The effect of wool velocity on the contamination of the wool in the first tank at steady state.

of three different purge cycles. In these simulations the tank begins clean of contamination at time $t = 0$, but with increasing time contamination settles down the bowl and concentration builds up, eventually reaching a quasi steady state. This is particularly clear in the simulation with the longest purge. The purge cycle for each simulation is shown beneath the graph and the effect of each purge can be seen as sharp decreases in contamination, as contaminants are flushed out. In the simulations, the more of the purge cycle that is spent purging, the lower the accumulated contamination in the tank.

Often, only the steady state concentration in a simulation is important, not what happens beforehand or the effect of the drainage pattern. To find the steady state concentration, a simulation is run until steady state

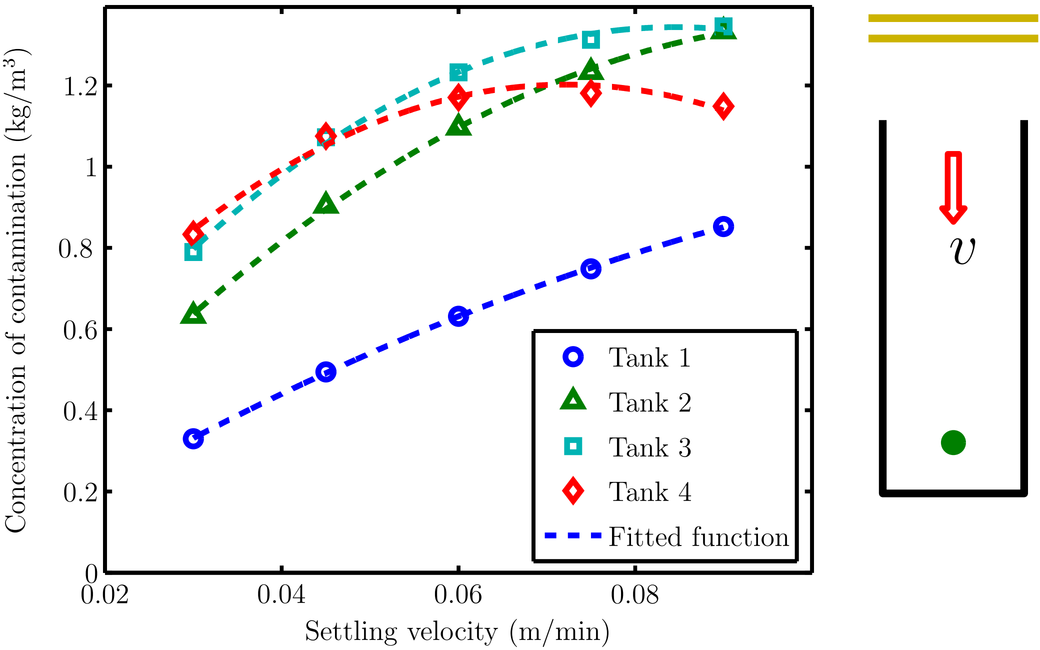

Figure 6: The effect of settling velocity of the contamination at the bottom of the four tanks at steady state.

is reached and then averaged over the last purge cycle. This is especially useful when assessing the effect of varying the parameters. Figure 5 shows the steady state concentrations in the wool region as the velocity of the wool w_{wool} is varied, with data from eleven simulations. At $w_{\text{wool}} = 0$, the contamination is zero as the wool is stationary and will eventually be completely cleaned. At the other extreme, as $w_{\text{wool}} \to \infty$, the wool is moving so fast that none of the contamination is cleaned and the contamination in the wool remains at the boundary condition, in this case $c = 0.4$.

Figure 6 illustrates the effect of varying particle settling velocity, showing the concentration at the bottom of each of the four tanks against settling velocity v_{set}. The fitted functions are best fit quadratics. Slower settling

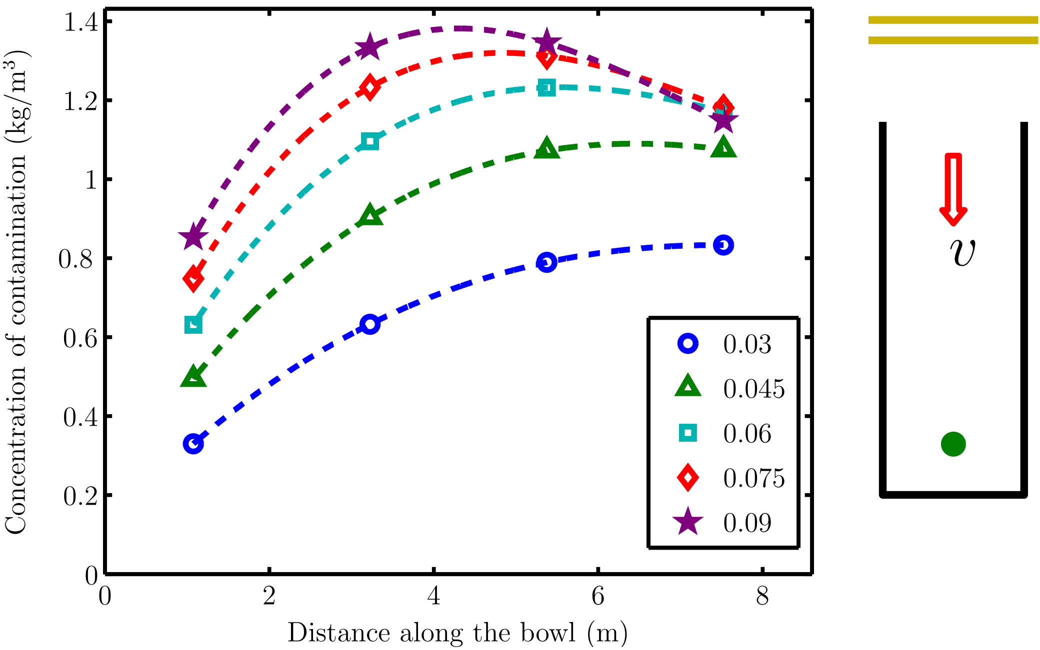

Figure 7: The contamination at the bottom of the bowl at steady state for different settling velocities.

particles are carried along by the cross-flow and settle in later tanks, while faster settling particles move quickly into the earlier tanks. So, each settling tank collects particles that settle at a particular speed more than others. For example, in tank four the optimal velocity is about 0.07 m/min. Figure 7 shows the same data as Figure 6, but arranged as concentration against distance along the bowl for different settling velocities. For all the velocities shown, the particles preferentially settle to one distance along the bowl and their concentration decays on either side. Particle settling velocity is linked to particle size, so Figure 6 could be seen as representing the distribution of particle sizes collected in each tank, assuming there is an equal amount of each particle size in the wool.

Some of the results from this model can be compared with another simple model of the wool in the scour bowl discussed in [2].

4 Conclusion

The movement of contamination, especially dirt, in a wool scour bowl can be modelled using discretised advection-diffusion equations, so as to give time dependent results, revealing, for the first time, how contamination builds up in the tanks and the effect of the drainage pattern. The steady state results show the relationships between the variable parameters and contamination in each tank and at different heights in the bowl. In particular, the relationship between particle settling velocity and contamination at the bottom of the tanks reveals the distribution of particle sizes that collect in each tank, which could be used to set appropriate operating parameters for wools containing different sized contaminants. The relatively complex model has revealed some simple relationships between the operating parameters of the scour machine and contamination throughout the wool and scour bowl.

References

[1] S. I. Barry, T. R. Marchant and G. N. Mercer. Grease recovery and dirt removal in wool scouring. *Mathematics in Industry Study Group, Proceedings*, ed: J. Hewitt and K. White, 2002. C22

[2] J. F. Caunce, S. I. Barry, and G. N. Mercer. A simple mathematical model of wool scouring. In Andrew Stacey, Bill Blyth, John Shepherd, and A. J. Roberts, editors, *Proceedings of the 7th Biennial Engineering Mathematics and Applications Conference, EMAC-2005*, volume 47 of *ANZIAM J.*, pages C34–C47, June 2006.

http://anziamj.austms.org.au/V47EMAC2005/Caunce2 [June 22, 2006]. C32

[3] D. B. Early. A theoretical analysis of equilibrium conditions in the continuous scouring of raw wool. *J. Textile Inst.*, **68**(2/3), 60–67, 1978. C22, C23

[4] G. F. Wood. Aqueous jet scouring of raw wool. Part IV. Continuous scouring—The approach to equilibrium. *J. Textile Inst.*, **58**, 49–53, 1967. C22, C23