Modal analysis of a small ship sea keeping trial

A. Metcalfe* L. Maurits* T. Svenson* R. Thach*
G. E. Hearn†

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Abstract

Data from sea keeping trials of a Scottish trawler are analyzed. The trawler sailed an octagonal course, each leg took over 20 minutes and data recorded twice a second. The natural frequencies of vibration for each of the six rigid body modes are estimated from the heave, surge, sway, pitch, yaw and roll time series. The time series are investigated for evidence of non-linearity. A time domain model is fitted to a roll time series, and second order amplitude response functions are then obtained using autoregressive estimators.

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*School of Mathematical Sciences, University of Adelaide, South Australia. http://www.maths.adelaide.edu.au/stats/staff/ametcalfe.html

†School of Engineering Sciences, University of Southampton, England.

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1 Introduction

We investigate the natural frequencies of motion of a 24 m Scottish trawler, during sea keeping trials conducted in a wide range of conditions in the North Sea. The reasons for performing sea keeping trials on these trawlers is that designs have changed, in response to new regulations, and they have become shorter, wider and heavier, and possibly overpowered and less safe [1].

The trawler sailed over an octagonal course. During each leg data were recorded every 0.5 s for over 20 minutes. Data were collected for all six components of motion of the trawler, for the wave heights, and for the wind speeds. Hearn et al. [1] investigated the amplitude response functions (gains) of wave energy to the heave and pitch motions of the trawler, with particular regard to the accelerations experienced in the bow. However, the remaining four motions and a modal analysis were not discussed.

Here we analyse the gains from waves and wind to all six components of motion, using the data collected by Hearn et al. [1]. In particular we attempt to detect natural modes of oscillation by comparing the peak frequencies in the H1 estimators of the gains. We also investigate the time series of the roll motion for non-linearity and estimate the second order frequency response function.

Table 1: Descriptive Statistics: wave, roll, pitch, heave, surge, sway, yaw.

Variable	N	Mean	StDev	Minimum	Median	Maximum
wave	2995	0.06632	0.49602	-1.78440	0.06300	1.84500
wind	2995	15.472	1.277	10.919	15.507	19.629
roll	2995	-2.2200	0.9608	-4.9200	-2.2000	0.9419
pitch	2995	2.2464	1.7560	-4.3770	2.2580	8.0100
heave	2995	0.020481	0.049094	-0.146600	0.020350	0.203400
surge	2995	-0.00378	0.23289	-0.82830	-0.01178	0.69640
sway	2995	0.00417	0.14995	-0.52580	0.00763	0.53600
yaw	2995	0.19615	0.29323	-0.80180	0.19710	1.07700

2 Analysis

2.1 Normal modes

The motion of a rigid body in a fluid can be described by displacements along orthogonal axes xyz and rotations about these axes. The displacements are surge, sway and heave along the x, y and z-axes respectively. The corresponding rotations are pitch, roll, and yaw. As there are six degrees of freedom, there will be six natural frequencies. However, there is coupling between the displacements and rotations and the normal modes are linear combinations of these [2, e.g.].

A summary of the time series of wave height (m), wind speed (knots), and the displacement (m) and rotation (degree) measurements made in the engine room, during leg 4, is given in Table 1 and Figure 1.

The substantial correlations between heave and pitch are expected [3, 4]. The smaller correlations, which are nevertheless statistically significant, between roll and pitch are not predicted by standard theory. An explanation for this is that heavy nets were loaded on the port bow and starboard stern, and these caused a noticeable corkscrew motion of the ship when it was

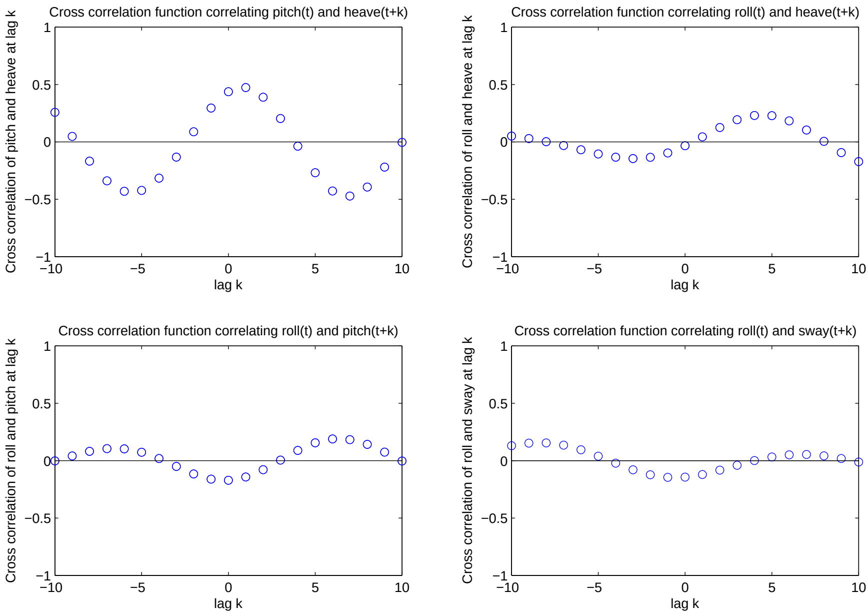

Figure 1: Cross-correlations: pitch-heave, roll-heave, roll-pitch, roll-sway.

under way.

Although we have not displayed the cross-correlograms there were correlations greater than 0.1 in absolute magnitude, at some lags, between: pitch and surge; roll and sway, roll and yaw; heave and surge, heave and yaw, and surge and sway. Overall, there is evidence of some slight coupling between most displacements and rotations, but that between pitch and heave is the most substantial.

2.2 Gain

The input spectrum and cross-spectrum were estimated by taking a discrete Fourier transform of the sample covariance function or cross-covariance function respectively. A Tukey window with truncation point of 200 was used in all cases [5, e.g.].

We chose to use the H1 estimate of the gain, absolute value of estimated cross spectrum to estimated input spectrum, rather than H2, square root of ratio of estimated response spectrum to estimated input spectrum, because it is unaffected by white noise in the response signal. A consequence of the definitions is that H1 is less than H2 at all frequencies. In Figure 2 we include both estimates of the wave to heave response, for leg 4.

For all gains, only frequencies below 1 rad/0.5 s were significant. We illustrate this using the gains for leg 4 of the octagonal course in Figures 3–6. Then we present the results for all legs, but only for frequencies below 1 rad/0.5 s.

The four gains of heave and pitch to wave and wind each have two peaks, Figures 7 and 8. These natural frequencies should be the same in each of the four gain plots and appear to be approximately 0.11 Hz and 0.16 Hz. The roll gains have a clear peak at about 0.09 Hz on the gain plots from wave and wind (Figures 7 and 8). This is a different frequency to the heave/pitch

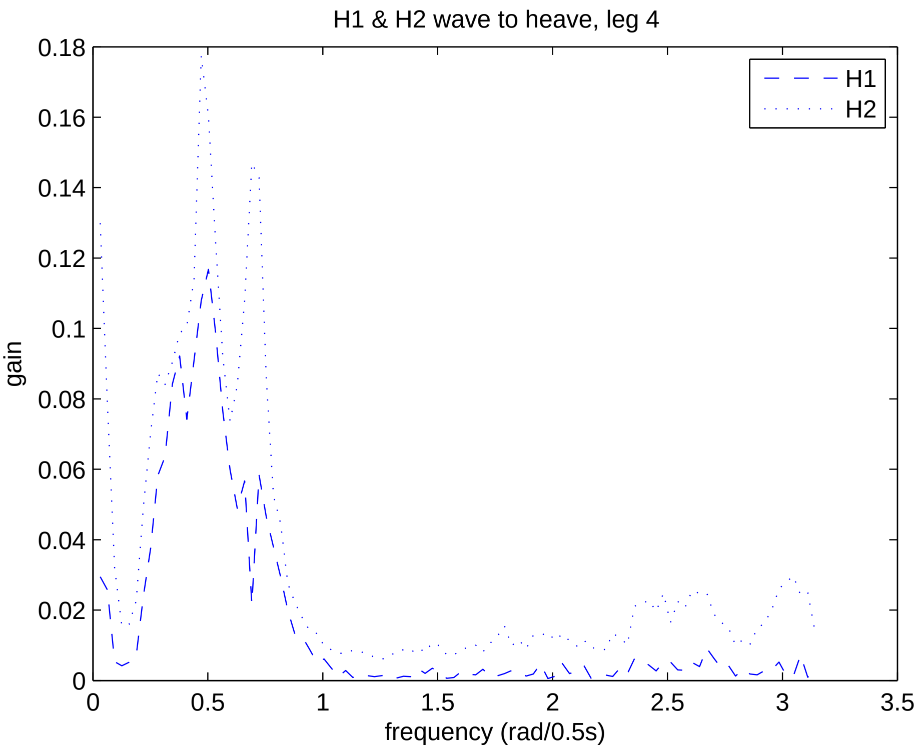

Figure 2: H1 and H2 gain estimates compared.

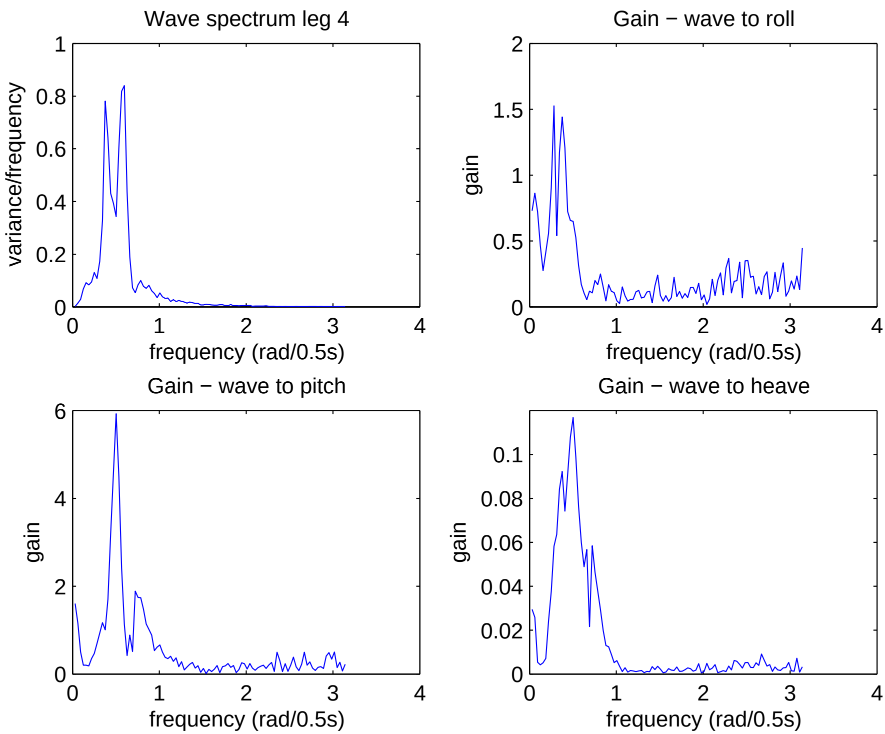

Figure 3: Leg 4 wave spectrum, and H1 gains of roll, pitch, and heave to wave.

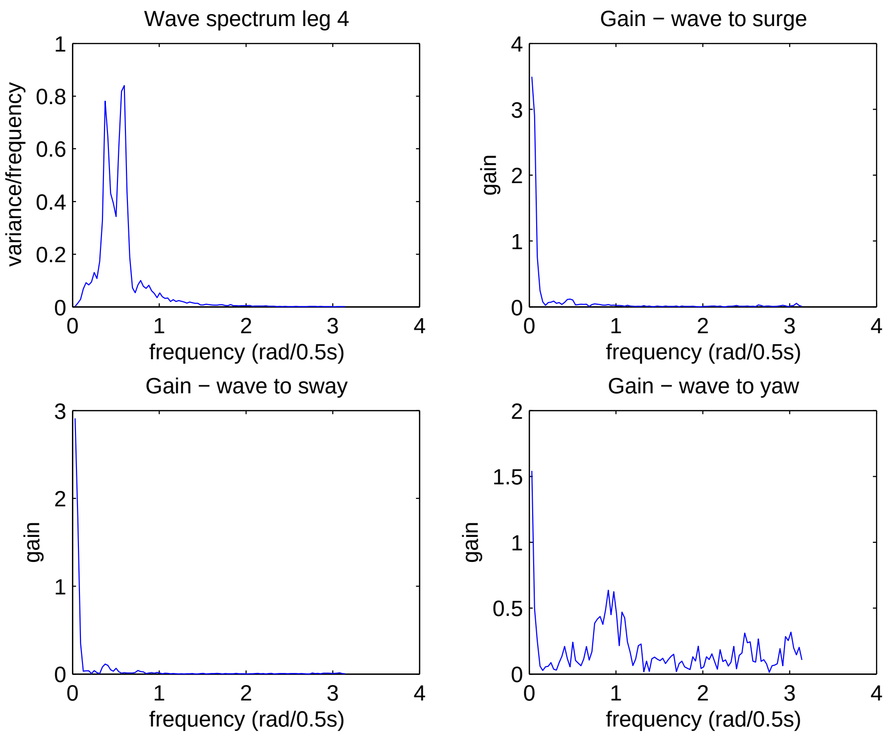

FIGURE 4: Leg 4 wave spectrum, and H1 gains of surge, sway, and yaw to wave.

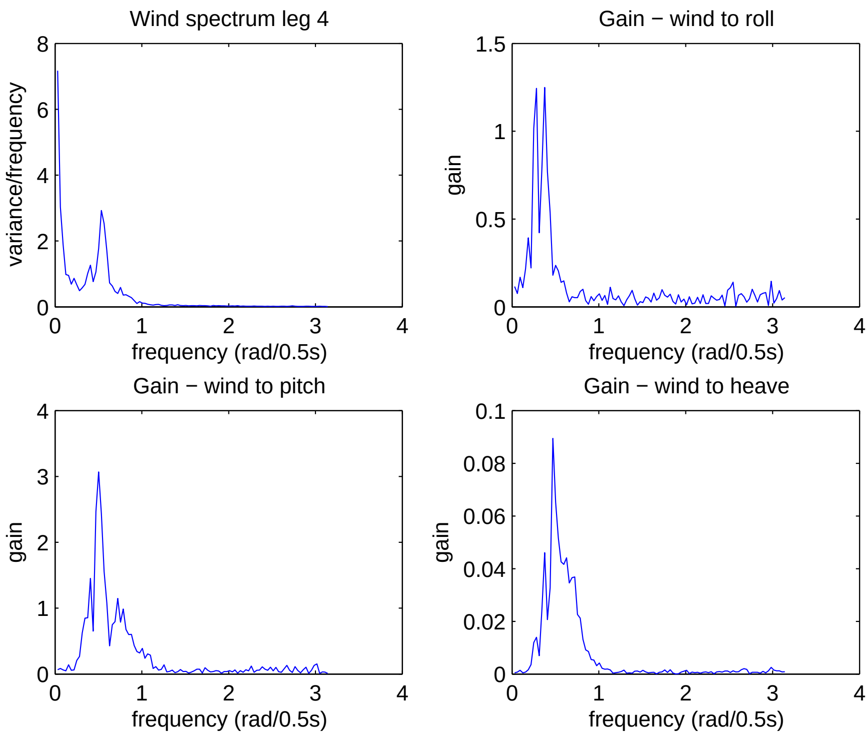

Figure 5: Leg 4 wind spectrum, and H1 gains of roll, pitch, and heave to wave.

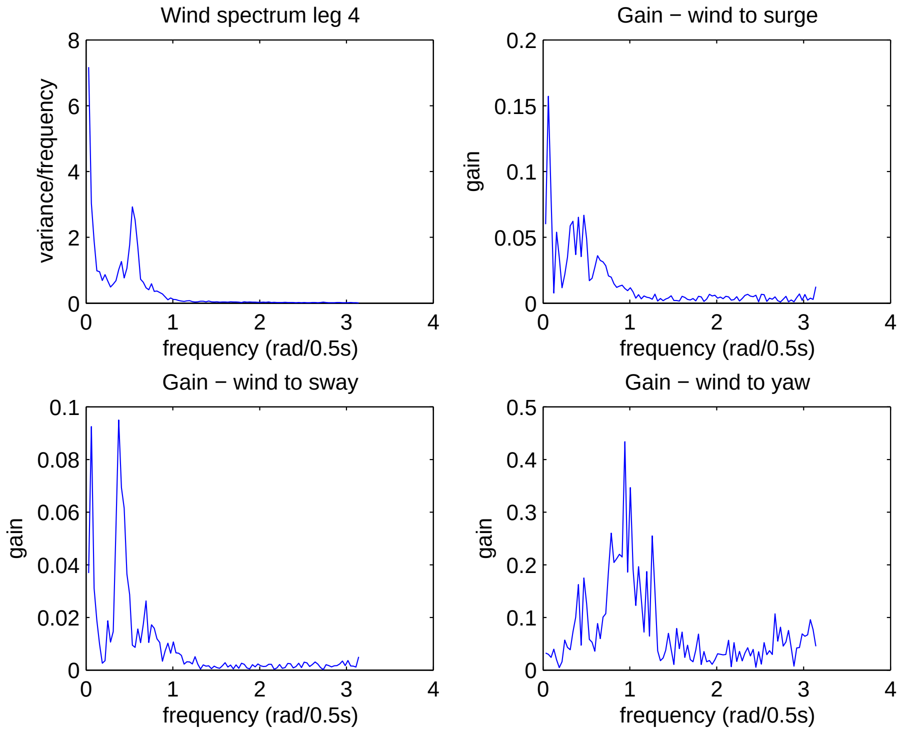

Figure 6: Leg 4 wind spectrum, and H1 gains of surge, sway, and yaw to wave.

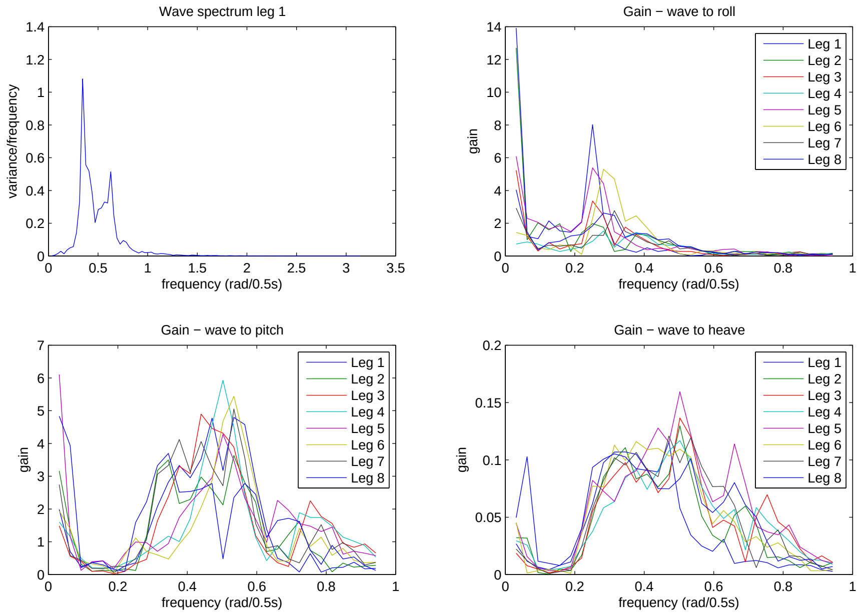

Figure 7: Leg 1 wave spectrum, and H1 gains of roll, pitch and heave to wave for all eight legs.

modes, showing that it is a different natural mode. Surge and sway appear coupled in the gains from wind (Figure 10) with one natural frequency at about 0.02 Hz and the other less precisely identified at about 0.12 Hz, the gains from wave (Figure 9) do not provide much information on these modes as they are not noticeably affected by the waves. There is little evidence of any coupling between roll and sway although some might be expected. The natural frequency associated with yaw (Figures 9 and 10) is hard to identify, possibly due to action taken by the helmsman. The slight evidence of coupling between the roll and pitch due to asymmetric loading of the vessel, provided by the correlations, is not apparent in the gain plots.

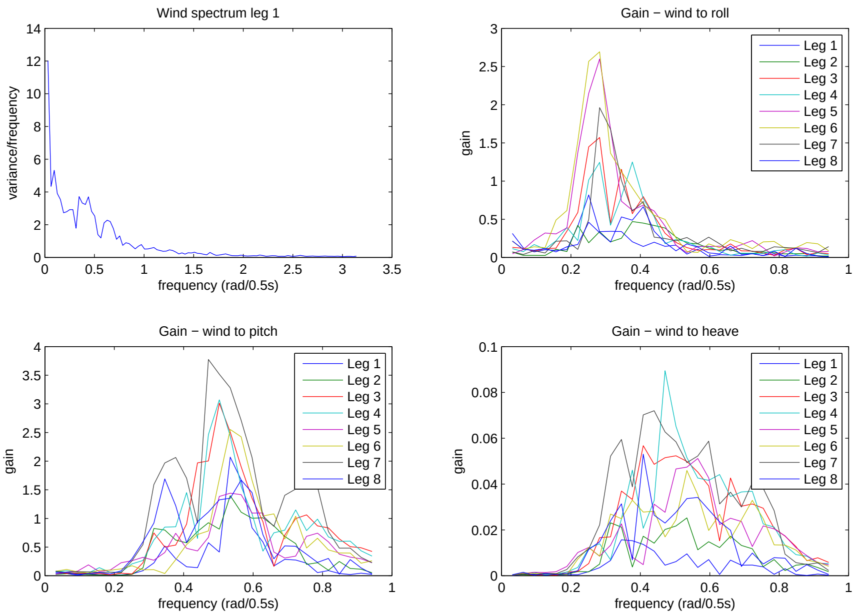

Figure 8: Leg 1 wind spectrum, and H1 gains of roll, pitch and heave to wind for all eight legs.

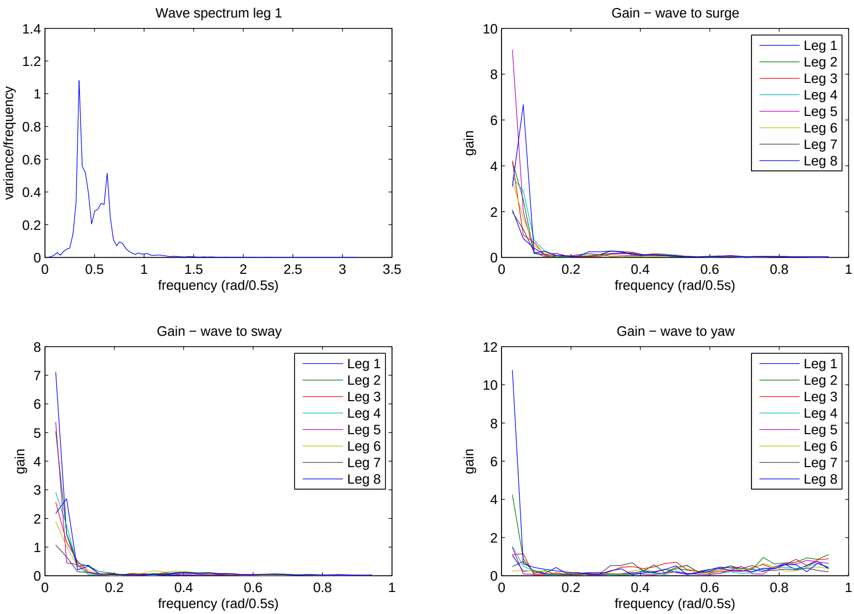

FIGURE 9: Leg 1 wave spectrum, and H1 gains of surge, sway and yaw to wave for all eight legs.

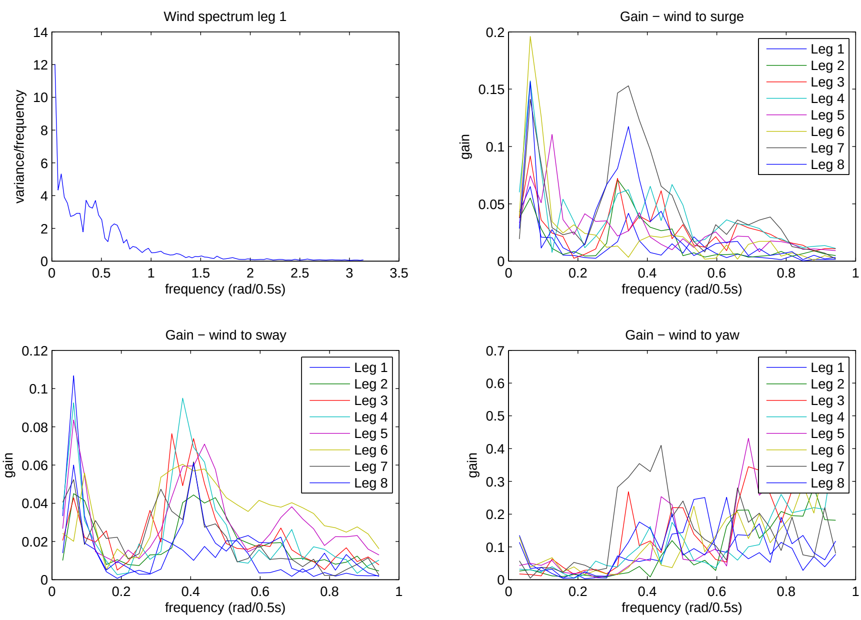

Figure 10: Leg 1 wind spectrum, and H1 gains of surge, sway and yaw to wave for all eight legs.

2.3 Non-linearity

There are several reasons why the hydrodynamic response of a ship will be not be precisely linear. In particular, the varying cross section of the hull accounts for non-linear buoyancy forces. The following model is typical of those for which we have found some justification for including a non-linear term. In the regression equation $y(t)$, $v(t)$ and $w(t)$ represent the roll response, wave height and wind speed at time t when sailing leg 4. All the estimated coefficients exceed twice their standard errors, and the coefficient of determination is 0.969.

$$\begin{aligned} y(t) \quad &= \quad -0.276 + 1.73\, y(t-1) - 0.789\, y(t-2) + 0.00567\, y(t-1)^2 \\ &\quad + 0.0588\, v(t-1) + 0.00692\, w(t-1) \end{aligned} \tag{1}$$

We use the probing method to fit first and second order response functions [6].

The roll response of Equation (1) can be expressed as

$$y(t) = a\, y(t-1) + b\, y(t-2) + d\, y^2(t-1) + c\, x(t-1)\,. \tag{2}$$

In this case the numerical values of a, b and d are the estimated coefficients in Equation (1): 1.73, -0.789, and 0.00567 respectively. The leading constant has been omitted as it is an offset which does not affect the dynamics. In this case it could represent a zero error or a list caused by wind loading. The input $x(t)$ in Equation (2) includes both wind and wave forces and is the sum of $v(t)$ and $w(t)$. This is valid since both $v(t)$ and $w(t)$ are general inputs. The numerical value of c is the sum of 0.0588 and 0.00692.

The system is probed initially with a single exponential input,

$$x(t) = e^{i\omega t}\,. \tag{3}$$

Then substitution of (3) into (2) gives

$$y(t) = H_1 e^{i\omega t}\,, \tag{4}$$

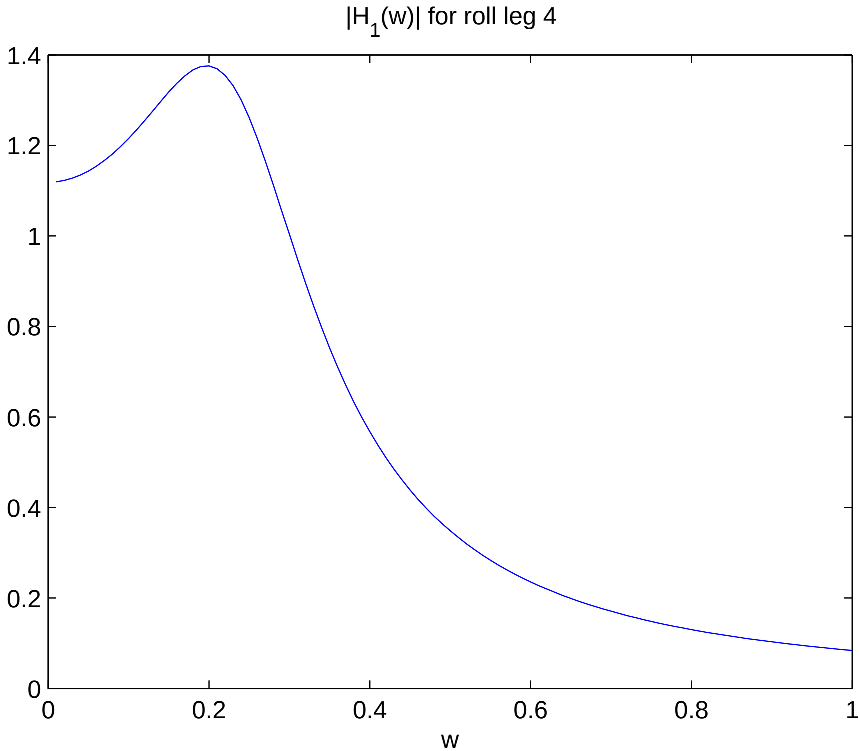

FIGURE 11: Amplitude of linear gain of roll from wave and wind, by frequency (rad/0.5 s).

where

$$H_1 = \frac{ce^{-i\omega}}{1 - ae^{-i\omega} - be^{-2i\omega}}\,. \tag{5}$$

Probing with two exponentials,

$$x(t) = e^{i\omega_1 t} + e^{i\omega_2 t}\,, \tag{6}$$

the output response has the form

$$y(t) \quad = \quad H_1(\omega_1)e^{i\omega_1 t} + H_1(\omega_2)e^{i\omega_2 t} + 2!H_2(\omega_1,\omega_2)e^{i(\omega_1+\omega_2)t}$$

$$+ H_2(\omega_1, \omega_1)e^{2i\omega_1 t} + H_2(\omega_2, \omega_2)e^{2i\omega_2 t} \,. \tag{7}$$

Substitution leads to

$$H_2(\omega_1, \omega_2) = \frac{dH_1(\omega_1)H_1(\omega_2)e^{i(\omega+\omega_2)}}{1 - ae^{-i(\omega_1+\omega_2)} - be^{-2i(\omega_1+\omega_2)}} \,.$$

We calculated $|H_1(\omega)|$ and $|H_2(\omega_1, \omega_2)|$ using the estimated coefficients in (1), and plotted them in Figures 11–12. There is a ridge corresponding to $|\omega_1| = |\omega_2|$ with a peak at a frequency of 0. The physical interpretation is that the square of the input signal has an effect and this will have two effects on the H1 estimate of the gain from wave or wind to roll: an increase in the response at 0; and a harmonic at double the natural frequencies which will appear as smaller peaks.

3 Conclusion

The natural frequencies are estimated to be: 0.11 Hz and 0.16 Hz, associated with the heave/pitch mode; 0.09 Hz associated with roll; and 0.02 Hz and 0.12 Hz associated with a surge/sway mode. We cannot make any precise estimate of a natural frequency associated with yaw.

Although linear models appear to give a reasonable approximation for the dynamic response of the trawler, at least for the amplitudes of oscillations occurring in these sea trials, we have evidence of non-linear effects which provides some explanation for the increase in the estimates of gains as the frequency approaches zero.

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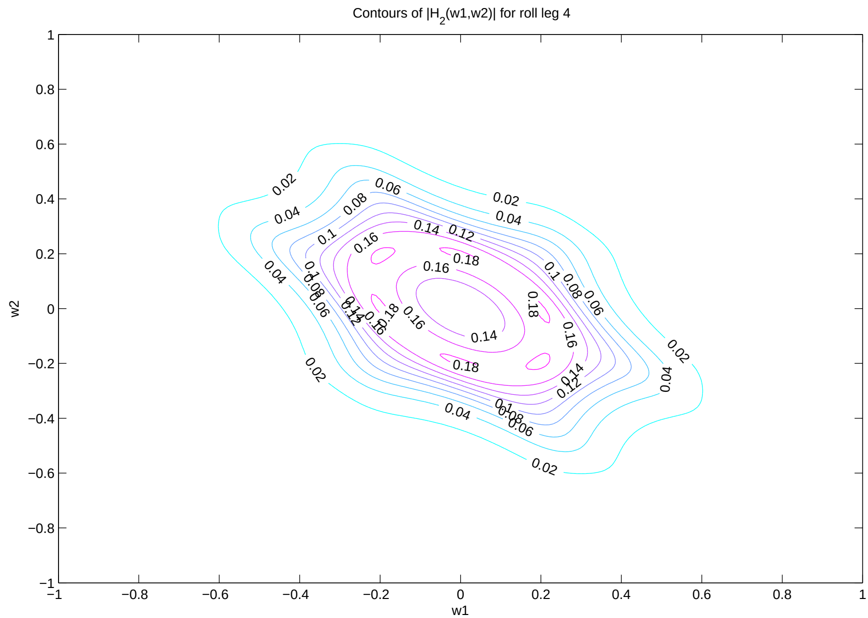

Figure 12: Contours (aqua 0.02 to pink 0.18) of amplitude of second order gain of roll from wave and wind, by frequencies (rad/0.5 s).

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