Proceedings of the 2022 Mathematics in Industry Study Group

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Abstract

This special Section of the ANZIAM Journal (Electronic Supplement) contains the refereed papers from the 2022 Mathematics in Industry Study Group (MISG2022) held at the University of Newcastle from 14–18 February 2022. This report provides the equation-free outcomes.

Contents

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1 Preface to the proceedings

This special Section of the ANZIAM Journal (Electronic Supplement) contains the refereed papers from the 2022 Mathematics in Industry Study Group (MISG2022) held at the University of Newcastle from 14–18 February 2022. The MISG is a special interest meeting of ANZIAM, the Australia and New Zealand Industrial and Applied Mathematics (ANZIAM) division of the Australian Mathematics Society. The MISG meetings take place annually and provide a forum where projects proposed by industry can be worked on intensively, by high profile scientists in the fields of Applied Mathematics, Statistics and Engineering, from Australia, New Zealand and the world beyond, along with representatives from the industries proposing the projects.

The writing of these papers was coordinated by the project moderators in consultation with the coauthors and company representatives. The manuscripts were submitted to the editors, Associate Professor Mike Meylan, Professor Ngamta Thamwattana and Professor Tony Roberts, and were subsequently refereed by two expert referees. On the advice of the referees, manuscripts were accepted for publication, subject to the recommended revisions, and formally approved by the editorial committee.

At MISG2022, two projects were presented from two companies, with 30 delegates participating online via Zoom and in person.

1.1 Industry Partners

We gratefully acknowledge the support of our industry partners:

- CarbonPump;
- Viper.

1.2 Acknowledgements

In addition to our industry partners, we gratefully acknowledge support from the following organisations:

- ANZIAM;
- Office of the NSW Chief Scientist & Engineer, Department of Industry, NSW Government;
- College of Engineering, Science and Environment, The University of Newcastle.

1.3 MISG 2022 Organising Committee

- Professor Ngamta Thamwattana (Co-Director)
- Associate Professor Mike Meylan (Co-Director)
- Mrs Juliane Turner (Administrative Support)

2 The Viper: Role of vibration in solid-liquid separation using a vacuum belt filter

Moderators: Phil Broadbridge, La Trobe University, and Kenneth Duru, Australian National University, Kyle Stevens, University of Newcastle.

Jord Industry Representative: Sam Caldwell.

During the early stages of processing and refinement, mineral ores are crushed and mixed with water to form a slurry. At some stage, the solids must be separated and the water returned to a pond. Improvements in solid mass fractions by 1% may have very large economic benefits.

One method used world-wide is the vacuum belt filter. A conveyor belt, typically 30 m long, conveys the mixture, typically for three minutes. During that time, a vacuum pump system reduces the pressure below the belt to around 0.7 atmospheres. The pressure gradient drives the liquid downwards through perforations. The solid particles are collected on a filter mat of around 2.5 mm thickness. In the data on crushed coal slurry provided by the company, the slurry enters at depth around 15 cm at 18% solids by mass. The material leaves as a well formed cohesive cake at 70% or more solid mass.

Jord has found that the final solid mass content can be increased to 75% if the newly formed cake, with negligible overlying water at around 16–18 m, passes under a 30 cm roller whose axle is attached to a Viper that vibrates at frequency 50 Hz and amplitude 2 mm. A mathematical model for this operation would lead to a better understanding of the process, and in the future, to a means of optimising its performance.

The solid/fluid mixture has different characteristics along three distinct segments of the belt. They are the super-saturated slurry region at the input, the unsaturated cake at the far end, and the emergent cake which is saturated over a few metres in the intermediate region where a thin film of water is observed above its surface. The Viper often operates in this region.

2.1 The Super-Saturated Slurry

There is a direct relationship for the conversion between solid mass fraction and liquid volume fraction. The latter is commonly used in porous media theory. At 18% solid mass fraction and typical relative density 1.6 for anthracite, the volumetric water content is above 0.85, well above the porosities of natural porous media. Therefore the mixture is a slurry with disconnected solid particles. As water is extracted downwards through the filter, a sediment is left behind and its thickness increases. That residue increases hydraulic resistance along the line. Assuming Darcy's law for flow through porous media, the water extraction rate was calculated. The matching hydraulic conductivity was within the range of a clay loam. This seems reasonable and it is some validation of the Darcy flow picture. The theory of saturated porous media applies until the form line, after which there is no more liquid overlying a formed cake.

2.2 The unsaturated formed cake

For most of the remaining belt beyond the Viper, there is no surface water and the cake must be unsaturated. The volume of additional extracted water is replaced in the cake not by surface water but by air. The larger pores in which the water-retaining forces of surface tension are weaker, are emptied first. This leaves connected channels of air as the paths of least resistance, for release of the pressure gradient generated by the vacuum system. Some of the energy of the system is wasted by transporting air. This problem is exacerbated by the formation of low-viscosity fingers. Since the viscosity ratio between air and water is only 0.1, the conditions are met for the Saffman–Taylor instability, and consequent formation of air fingers that are wider than pore sizes. The remaining water is held in finer pores that result in lower hydraulic conductivity. Between the form line and the end point, the volumetric water content reduces from around 0.5 to around 0.4 without the Viper and 0.33 with the Viper. Over that range the hydraulic conductivity of common soils reduces to less than 0.1 of saturated conductivity. Relatively little more water is extracted from the last 8 m of the belt.

2.3 The Working of Roller and Viper

Every part of the newly formed cake passes under the vibrating roller for 0.4 s, over a contact distance of 5 cm. The improvement in solid mass fraction measured at Station 3, 18 m along the belt, within the formed cake zone and just beyond the roller, is equivalent to additional removal of 2 to 3 mm of water depth. If the roller at its highest elevation is still touching the cake, then it is 4 mm deeper after one half cycle. All solids are retained in the cake so this reduction can be compensated only by shrinkage of the cake or by displacement of water by air. However, this cannot be the whole story since it does not explain the observed frequency dependence. Water extraction is more efficient at 50 Hz Viper frequency than at 15 Hz.

With an amplitude of $a = 2$ mm, and at frequency 50 Hz, the peak acceleration due to vibration is approximately 13 g. This is adding to gravity, causing oscillations in the capillary rise. At the microscopic level, a balance between surface tension and gravity and other inertial forces could occur only if some water left the pores. At the mesoscale, our calculations show that the induced stress due to inertial forces exceeds the minimum for liquefaction to occur. During contact, seventeen full oscillatory cycles occur, again exceeding the second criterion that ten cycles are needed for liquefaction. This conclusion is supported by calculation of the very low Deborah number.

At 15 Hz, only six vibrations take place. Although the general theory of liquefaction is not fully settled in the scientific literature, the most common theory indicates that the frequency of 15 Hz should be marginal for liquefaction. However, the Darcy flow model indicates that over the time of contact, liquid extraction is significantly improved even if only a minor upper portion of the cake is liquefied. Darcy's law can be used to infer the approximate thickness of the unbroken sediment pile beneath the liquefied layer. Over that liquefaction time, the thickness of the sediment pile is reduced by 2.6 cm. That reduction would be proportional to the number of cycles of vibration. If the frequency is reduced from 50 Hz to 15 Hz then the sediment pile should be reduced by 8 mm rather than 26 mm. In that case, Darcy's law predicts an extra outflow of depth 0.9 mm compared to 2.2 mm at 50 Hz. After the shake-up, the disturbed upper levels of the cake would reassemble either in a saturated state that would be more compact due to the loss of water volume, or in an unsaturated state with additional air.

2.4 Conclusion

The data show that adding a second Viper does improve water extraction, but only by a relatively small amount. At a short distance past the first Viper, in the region of re-consolidation, the matrix structure is more cohesive. The conductivity decreases further along the belt and exiting water flux decreases. If a second Viper is to be used, then it should be placed soon after the first, preferably in phase, in the early part of the formed cake.

Increasing belt speed does increase throughput. However, the data show that not so much water is removed when the speed is increased from 14 cm/s to 20 cm/s. This may be due to reduced contact time with the roller.

The only feasible explanation of the Viper frequency dependence of water extraction, is partial liquefaction of an upper portion of the cake, over a short time of roller contact. During that time, downward water flow and extraction are improved. A higher frequency is better because it causes higher stresses but eventually the period of a higher frequency (not yet determined) may be shorter than the response time of the cake, just as in a damped harmonic oscillator beyond the resonant frequency.

We recommend to visually inspect the region near the roller for turbidity and any evidence for partial liquefaction.

Then quasi-1D flow model underestimated the cake thickness by a few mm and it underestimated the solid mass fraction at Station 1. More work could be done on a more sophisticated 2D flow model in both slurry flow and Darcy flow, with hydraulic conductivity decreasing with distance, as clogging occurs.

3 CarbonPump: Improving the design of the SuTiT for soil core sampling

Moderators: Melanie E. Roberts, Griffith University, Fillipe Georgiou, University of Newcastle, Edward J. Bissaker, University of Newcastle.

The collection of soil cores is essential for the evaluation of soil carbon stocks, and hence is a necessary component of soil carbon trading. Soil cores are typically collected by driving a cylindrical tube as much as 1500 mm into the ground, which, when extracted provides a soil sample across multiple soil horizons. There are many complexities in soil core sampling. A sampling rig must be suitable to obtain samples across a diversity of soil types and properties including sandy soils (which risk falling out of the tube as extracted) clays (which when wet stick to the inner walls of the tube), and rocks (challenging to drive the corer through). Furthermore, the speed of sampling (including processing of samples) affects the cost of obtaining samples.

While there is some diversity in approaches, in dry-land conditions typically two approaches are used to collect soil samples: a single walled tube driven into the ground, and a lined tube where the inner tube is extracted with the sample for processing. In both cases the tubes are forced into the ground using a combination of force (hydraulic ram and mass) and impact (jackhammer). The pros and cons of these two approaches depends on the sampling depth, accessibility of the sampling site, and the soil composition at different depths.

CarbonPump favour a double-tube approach for ease of deployment in the field. They investigated a number of different approaches before custom developing the SuTiT, a flexible, removable steel inner tube. The advantages of the SuTiT over alternatives include ease of use in the field, both for deployment and processing of samples, reduction in waste over single-use options, and depending on the lifespan of the SuTiT a reduction in cost. This MISG focused on improved understanding of the mechanical properties of the SuTiT with the aim of improving the design to increase the life-time of each tube.

3.1 Problem Description

The SuTiT (Figure 1) is an inner tube made from steel and consisting of three regions in the axial direction and five regions in the radial direction. The base (and top) of the SuTiT has a series of rectangular cutouts spaced around the radius. These cutouts work to anchor the SuTiT to the collar, preventing it from moving within the outer tube during deployment or extraction. A soil retainer is positioned between the collar and the SuTiT to prevent samples from falling out during extraction. The central region of the SuTiT, which occupies the majority of the length, is a combination of pressed and kerf-cut regions.

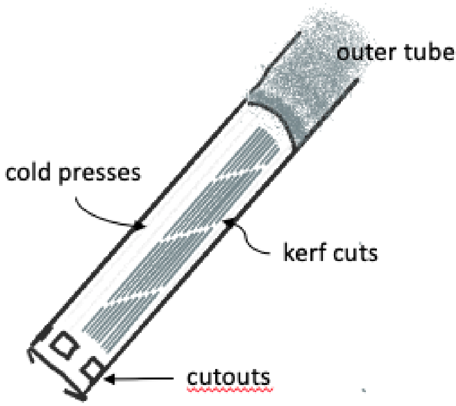
At rest the SuTiT has a natural trough or U-shape, achieved through cold presses at approximately 1/3 and 2/3 along the short axis creating the corners (Figure 1). Kerf cuts are used at the edges and centre to provide flexibility. This combination allows the SuTiT to be easily bent into a roughly circular shape for insertion into the outer tube, while returning to the trough shape on extraction for ease of access to the core sample.

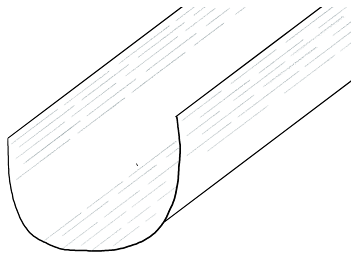
While the SuTiT meets CarbonPump's operational requirements, they have observed opportunities for improvement. The life-span of the SuTiT is unpredictable, due to failure or fracture when under load, which compromises the cost-effectiveness of the product. Furthermore, the laser-cut kerf regions account for the majority of the production costs. As the kerf-cut approach has to date been explored by trial and error CarbonPump are interested in understanding if design improvements could improve durability and/or reduce production costs. This MISG focussed on the flexibility and durability of the central length of the SuTiT.

The focus of this MISG was therefore to

- Understand the trade offs between flexibility and strength of the SuTiT given the combination of cold press (plastic bend) and kerf cut (elastic bend) regions.

Figure 1: Schematic of the SuTiT inner liner. The left figure shows the SuTiT inserted into the outer tube (viewed from above). The right figure shows a slice of the SuTiT to illustrate the locations of the kerfed region. Cold presses are used to bend the flat metal sheet into the U-shape at rest. Kerfing (indicated by the grey lines) on the upper edges and along the base of the tube provide flexibility. The kerf pattern indicated is illustrative only.





- Understand implications of the kerf pattern on the strength, flexibility and durability.
- Where appropriate, make recommendations for design alternations to improve the durability and production cost of the SuTiT

3.2 Approach

The team approached these challenges using a combination of literature review and mathematical modelling. The literature review focused on identifying existing knowledge on the relationships between curvature (flexibility) and strength in kerf bending, and fractures in metal structures subject to kerf-bending and metal origami. The mathematical models focused on force-deformation in thin steal beams.

3.2.1 Learnings from the Literature

Kalama, Tzoni, and Symeonidou (2020) investigated curvature and strength in MDF samples of a variety of kerf patterns using an experimental approach. They considered patterns that provided both single and double curvature. Patterns that exhibit double curvature are not relevant to this study, we therefore focus on the single-curvature patterns. In general, their study found that the greater the density of cuts (more product removed) the greater the flexibility (smaller radius of curvature) but at the expense of structural strength. They also investigated the effect of discontinuities in the pattern on structural strength. Their study showed that the points where the lines present discontinuity are more strained during bending and are the most likely sites for fracture points. While this study was conducted using samples prepared in MDF, we expect that the general conclusions are applicable to our case.

The relationship between spacing between cuts and bending was also explored by Güelci, Bacinoglu, and Alaçam (2016). They also found that less space between cuts lead to greater bending. However, they determined that the shape of the cut has an effect on the total amount of bending, with the direction of the cut introducing anisotropy in the bending even with the same cut density.

3.2.2 Mathematical Modelling

A combination of mathematical models were used to explore the relationship between strength and flexibility in the case of the SuTiT. These models are underpinned by observations from the provided prototypes. The shape of the tube under different applied forces was extracted by taking stamps of the end of the SuTiT when bent by hand into different curves. These stamps were digitised and then turned into piece-wise continuous curves using a cubic spline interpolation

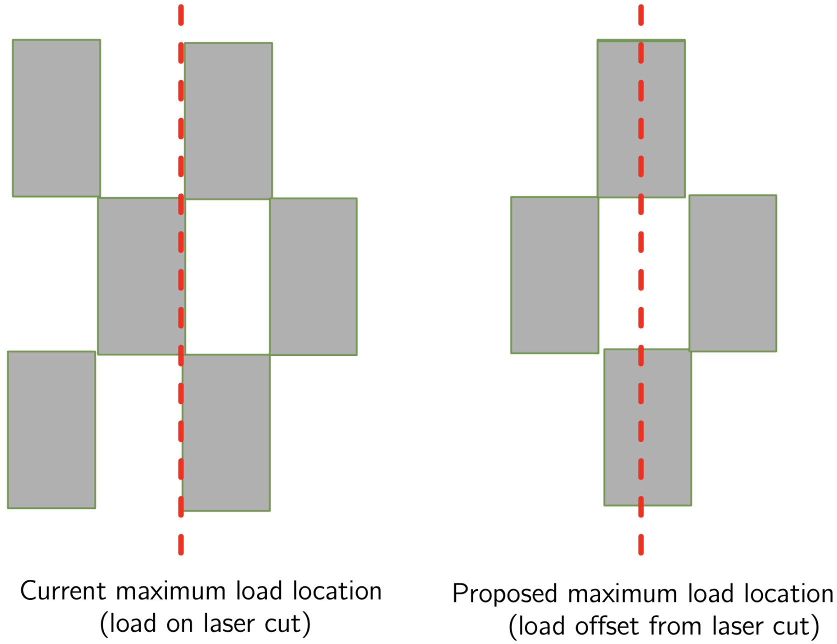
As a first approximation, a simple linear model for the deformation of a thin beam was explored. This model considered small deformations from the U-shape by dividing the tube into five regions. The model assumes the tube is symmetrical about the centre-point and allows the kerfed base, cold pressed region, and kerfed edges to have different stiffness coefficients (high stiffness is equivalent to low flexibility). We assumed the kerfed regions had greater flexibility than the cold-pressed region. This study has two key results:

- Even with very high levels of flexibility, the edge regions would not approach a circular shape and appear almost linear; and
- The kerfed base region is responsible for the majority of the deformation achieved.

Examination of the SuTiT prototypes corroborated these results. A consequence of the near-linear shape at the edges of the SuTiT is that a point force arises while inserting it into the outer tube. A corresponding and opposite force arises at the base of the SuTiT. As the kerfed base will achieve a near-circular shape, this creates a high friction point through that region, which could explain the wear patterns observed on some tubes. As a perfect circle will not be achieved, we further recommend that an odd number of kerf cuts is used across the base region such that a fold line (laser cut at high stress point) does not arise and so that the high-stress is distributed over a larger area see Figure 2.

Two other approaches we investigated include a calculus of variation approach and a an elastic theory approach, although these approaches did not yield additional results during the MISG. The complete calculus of variation theory was applied to understand non-linear effects and test the suitability of the 'small deformation' assumption of the linear theory. This model considers the effects of curvature more generally. Initially, a constant flexibility throughout

Figure 2: Odd and even kerf patterns with corresponding offset of laser cut from maximum load position (red line).

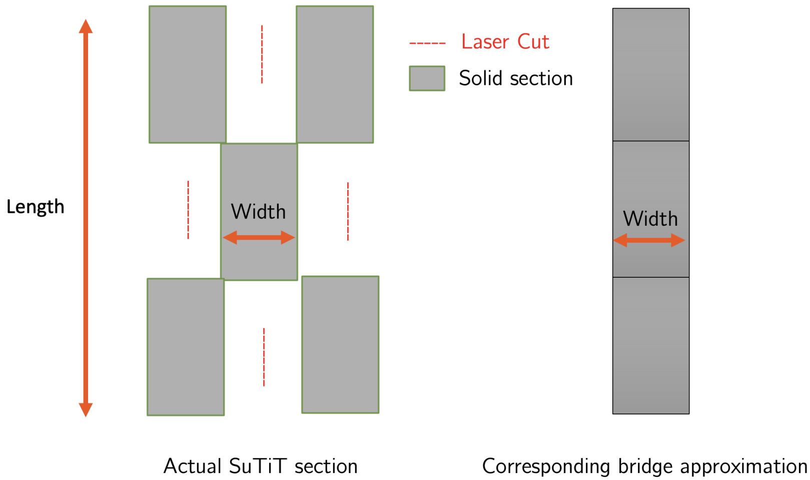


the circumference of the tube was assumed. Further relaxing our assumptions, we also considered the full elastic theory of a near-circular ring under a radial force. In contrast to the linear model, which assumes that a force is applied at the edges of the tube, this model considers the forces associated with squeezing the tube. This squeezing model reflects the forces applied by the outer tube to the elastic inner tube.

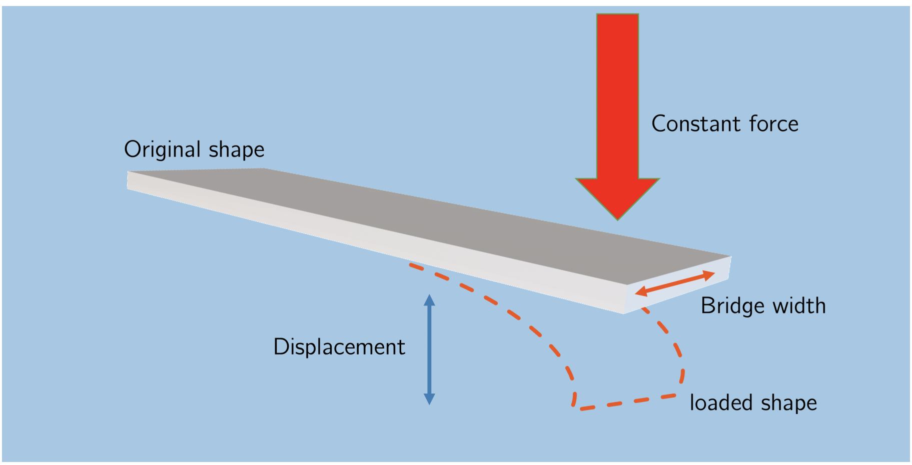
The above models consider the kerfing effects through bulk parameters, but are unsuitable to understand the effects of different kerf patterns without a model to relate the pattern to these parameters. We used two approaches to explore kerf patterns.

Figure 3: A, SuTiT kerf section and corresponding equal width bridge approximation. B, Constant load FEM deformation approach for varying width, constant thickness and length.

A



B



A Finite Element Model (FEM) was constructed to explore the effect of the bridging width on the deformation of a thin beam under an applied force (Figure 3). Given the relationship between strength and flexibility, CarbonPump should seek to limit flexibility only to what is necessary to achieve the desired shape. Observations from the prototypes indicates that more flexibility is achieved than required, which not only sacrifices strength but also increases the production costs. Our FEM simulation utilises a bridge approximation assumption, where we assume that the kerf sections behave approximately as a single bridge with corresponding width, as outlined in Figure 3A. Our model shows that as the width increases, the maximum displacement decreases. Hence the model suggests to account for the SuTiT's over-flexibility, a laser cut length reduction of 50% can be applied without significant impact to the SuTiT's ability to be placed into the outer tube. This reduction in laser cuts required, and a corresponding increase in strength should reduce the SuTiT production costs and increase longevity.

We further explored the kerf pattern using a static equilibrium model for the forces at the bridges while under load, that is while bent into the circular shape. This model shows that the angle between successive bridges (Figure 5) is important for minimising the stress at the connection points, which have been observed to fracture. Bridges arranged in lines at an angle to the applied force will experience a twisting force, increasing their risk of fracture. To counter this, bridges should be arranged in lines perpendicular to the applied force, that is parallel to the short axis of the SuTiT. However, this will make the SuTiT slightly harder to bend.

3.3 Recommendations

The strength and durability of the SuTiT could be improved by exploring the following:

- The kerfed region at the edges results in a near linear shape while adding little to the flexibility of the tube. CarbonPump should consider

Figure 4: FEM simulation of maximum displacement change bridge width relationship. Model predicts a reduction of maximum displacement by 50% (approximately), and hence to double bridge width.

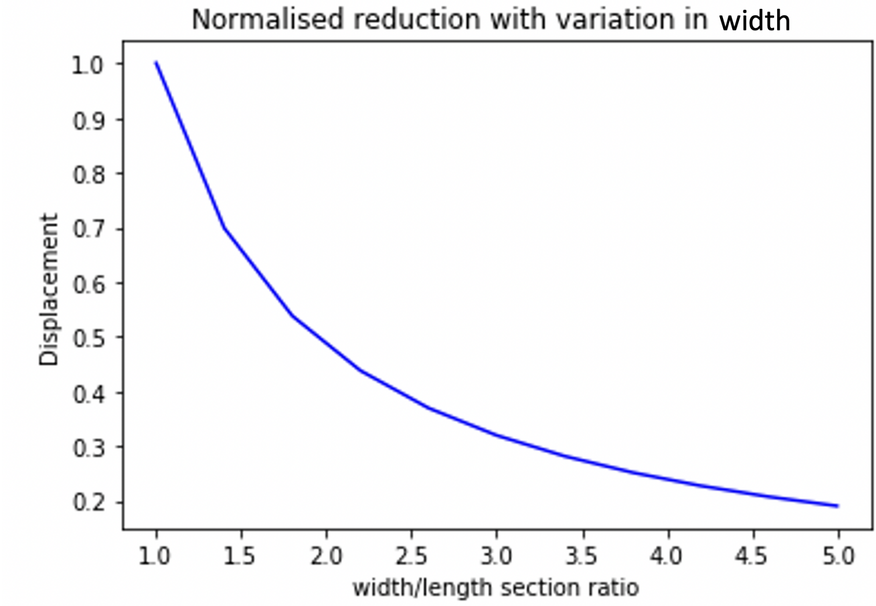
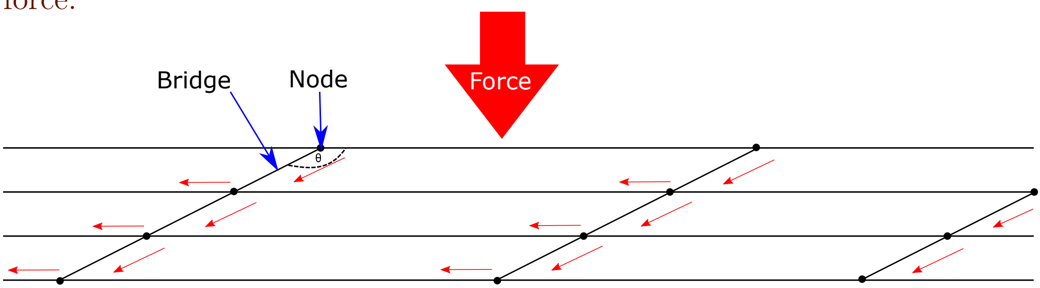


Figure 5: Method of static equilibrium. By using this method we found that the force occurring over the bridge is increased proportional to the angle between bridges, in addition by having the bridges offset it induces a twisting force.



increasing the width of the kerfed region at the base and use cold presses to obtain the circular shape.

- Bridges arranged in lines at an angle to the applied force introduce a twisting force at the bridges. CarbonPump should consider using kerf patterns where the bridges are perpendicular to the applied force.
- A trade-off exists between flexibility and strength. CarbonPump should seek to minimise the amount of material removed (number of cuts) to achieve no more flexibility than that required. Our models suggests a laser cut length reduction of 50% will maintain appropriate elasticity for outer tube insertion and reduce manufacturing costs.

Acknowledgements The moderators thank the industry representatives, Ignatius Verbeek and Stuart Morris. We acknowledge the hard work and contributions of the group members: Barry Cox, Jim Hill, Michael Meylan, Josiah Murray, Bishnu Lamichhane, Winston Sweatman, and Ngamta Thamwattana.

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