

Video analysis to understand e-learning of vector calculus

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Abstract

At RMIT Mathematics, we have been using Computer Algebra Systems Maple or Mathematica in undergraduate teaching for more than 15 years. Many of our courses use computer algebra systems in some support mode for what are otherwise traditional courses. However a few courses have been developed and run in a Maple “immersion” mode where Maple is used for class presentation, computer lab classes and assessment entirely with Maple. All course materials (including the examination) are delivered electronically over the web (using the course management software BlackBoard). This is a fundamental

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change in the paradigm by which we teach and learn mathematics. Teaching materials (Maple files) have been developed for a vector calculus course. Technology is used increasingly in teaching mathematics at all levels and studies show that mathematics learning is enhanced by using technology. However evaluations have been limited. Video analysis methodology is well established and utilized in large international studies in school settings but has not been used for undergraduate mathematics, much less for computer algebra immersion mode teaching and learning. Computer mediated learning is highly visual in form, but, unlike conventional instruction, generates no written paper record that might be collected and analysed. The adaptation of existing video analysis techniques offers new possibilities for the investigation of learning in such new undergraduate computer laboratory settings. The teaching and learning environment is intrinsically electronic and includes some elements of new e-learning which do not exist in a paper and pencil environment. As an example, animation is used to illustrate the generation of the 2D domain for double integration. Video captures both the visuality of the medium of instruction and the dynamics of the connections between representational forms, both by the instructional medium and by the learner. The research design for this study was deliberately exploratory in character: both in terms of understanding the new e-learning occurring and in the adaptation of the techniques of video-based data collection and analysis.

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1 Introduction

Technology such as Computer Algebra Systems, CAS, is being used increasingly in teaching secondary and tertiary mathematics. Many studies [1, 12, 16] show that learning is enhanced by using calculator and computer technology. CAS has been used in undergraduate mathematics education for about fifteen years: typically to support the learning of otherwise traditional courses. This support mode usage could accurately be described as using CAS in a super-calculator mode. CAS use at the secondary level in Australia (Victoria), the USA, Denmark and France is typically with CAS calculators and even the widespread use of CAS in Austria uses Derive in a super-calculator mode. There is a vast research literature about secondary and tertiary mathematics teaching using CAS in super-calculator mode (see the reviews [12, 13, 14]). This project initiates a study of the nature of the learning products when the instructional program moves beyond this super-calculator mode into true e-immersion.

At RMIT Mathematics, several innovative e-mathematics courses are conducted in a Maple immersion mode. The Computer Algebra System, Maple, is used for all presentation, teaching and assessment [3, 4, 5, 6]. Teaching is face-to-face in the computer laboratory. Students download all Maple files for teaching, assignments and examinations from the web. All student work is carried out, submitted and returned with marking as Maple files via the web. This is a fundamental change in the paradigm by which we teach and learn mathematics. The Maple immersion mode of teaching and learning of tertiary mathematics is unique to these courses at RMIT. The learning is different when totally CAS immersed and this CAS immersion learning has not been researched. The Maple immersion mode of teaching and learning is now sufficiently mature that creative and rigorous research into the learning

is timely. Given the high overhead of staff resources (about 10 hours preparation for each hour in the class or computer lab), research into the teaching and learning would be most valuable. Structured, careful research, using the most sophisticated technology, should provide significant insights into this new teaching and learning mode and into the novel e-learning it produces.

The videotaping and analysis methodology offers an innovative and comprehensive research tool [8, 9]. Several research questions can be addressed, such as What should be the balance between presentation and directed student work? What engages the students as active and cooperative learners? How can deep learning be achieved? What is the role of visualization in conceptual development?, and the data re-analysed to investigate further questions not yet posed. The effectiveness of this approach (and the multiple analyses) is seen in the research text edited by David Clarke [9]. Video analysis techniques have been used in large and small scale studies, locally nationally and internationally at primary and secondary school levels, but not, to our knowledge, for undergraduate studies. We propose to initiate the transfer of research expertise with video data collection and analysis methods to the tertiary level. This article reports on a project that investigates the viability of adapting a proven methodology to a new setting. This project is the commencement of a larger proposed study to conduct sophisticated research into technology facilitated learning of tertiary level mathematics by video-taping and analysing Maple e-mathematics classes (in the computer laboratory). The long term aim is to understand and hence optimise the learning of tertiary mathematics in such technology rich settings. The learning theory from this larger study will benefit the tertiary sector directly as well as having potential to benefit the school system as CAS use moves past the intermediate use of CAS calculators (as in the new course in schools in Victoria, Australia: VCE Mathematical Methods (CAS)) to the powerful computer based Maple or Mathematica.

It is clear that traditional assessment strategies are inadequate (paper based modes being poorly aligned with contemporary mathematical activity).

Old assessment is blind to new forms of learning. New assessment for new forms of learning is required and new forms of assessment are now possible [7]. However, there is a need to understand this new form of mathematics learning (e-learning) in order that assessment might better reflect the nature of the new learning and the new mathematical practices being developed.

The proposed study program will contribute to transforming the paradigm by which tertiary mathematics is taught, learnt, assessed and carried out. To enable and inform this transformation, it is essential to determine the processes and products of e-learning in the sort of innovative e-immersion setting that is the focus of this study. This article reports on the first step: in the second semester of 2005 we videotaped six hours of laboratory classes and three video-stimulated student interviews of a third year vector calculus course run in Maple immersion mode.

2 CAS: Internationally and at RMIT

Technology in the form of calculators and computers has been used widely in mathematics education and has a vast research literature. A key development was the reform calculus movement originating in the USA, led by the Harvard Consortium (supported by a National Science Foundation grant) which did much to promote the “rule of three”: Every topic should be presented geometrically, numerically and algebraically [11, p. vii]. The development of graphical calculators provided an easily accessible technological tool for numerical and graphical computations. It would appear that the use of graphical calculators has been widely taken up in high schools and to a lesser extent in lower levels of college or university mathematics. As an example of this approach, see Hughes-Hallett et al. [11] which is the first edition of the text written by several of the leaders of the reform calculus program. Traditionalists might well find that this book is almost a traditional introductory calculus text with a sensible and not very subversive incorporation

of the “rule of three” approach. From the comment on Technology in the Preface [11, p. viii]:

We take advantage of computers and graphing calculators to help students learn to think mathematically. For example, using a graphing calculator to zoom in on functions is one of the best ways of seeing local linearity. Furthermore, the ability to use technology effectively as a tool is of itself of the greatest importance. . . . Test sites have used the materials with graphing calculators, graphing software, and computer algebra systems.

The use of CAS as a pedagogical tool is now an important issue in many countries and for both the upper secondary and tertiary mathematics education sectors (for reviews, see [1, 10, 12, 13, 14, 16]). Many international conferences on mathematics education have a focus on technology: for example, the International Conference on Technology in Collegiate Mathematics, ICTCM, had its 17th annual conference in 2004. All reported studies use CAS in a support mode. Activities include pencil and paper tasks. Amongst other things, this presents some serious issues about assessment and what role the CAS should and should not play [12, e.g.]. Public examination systems tend to either try to set CAS neutral questions (such as the Baccalaureate Generale, France) or have a technology free examination and a CAS permitted examination (Baccalaureat, Denmark; recommended from 2006 for VCE, Victoria, Australia) and some variations on these in Austria, Switzerland and for Advanced Placement in the USA. At the tertiary level, a famous exception is the course by Jerry Uhl et al. *C&M: Calculus and Mathematica*¹ (which has run successfully, since 1989, in parallel with a traditional lecture group). See Jerry Uhl’s inspiration article “Why (and how) I teach without long lectures”. In this article Uhl compares his pedagogical method to that of studio learning. David Smith [16] regards

¹See <http://www-cm.math.uiuc.edu/>

C&M, one of the most successful products of the calculus reform initiative. By “successful”, I do not mean in the commercial sense . . . a number of research studies . . . compared *C&M* to traditional courses and found significant learning gains for the *C&M* students.

The *C&M* courses are the only close relatives to the Maple immersion courses that are the focus of this project but there are many differences in pedagogical approach (and personnel), curriculum and in the CAS environment employed (*C&M* uses Mathematica). The instructional program that provides the setting for this project makes innovative use of Maple as the universe of discourse: all presentation, learning and assessment occurs within a Maple environment. Paper is not used — the only non-Maple element is the use of the package BlackBoard as a web course management system (for course outlines, exchange of Maple files and e-gradebook) and communication (course announcements and e-mails). At RMIT Mathematics, Maple immersion courses are being actively developed as well as materials to support other courses. This approach can be developed for online delivery. In addition, a variety of assessment approaches are being explored which use Maple — including computer aided assessment where Maple is used to evaluate the answer symbolically (exactly!) [7]. The Maple immersion mode of teaching undergraduate mathematics is innovative and unique. This new pedagogical approach (where the universe of discourse is Maple) is in urgent need of rigorous research into the learning and the development of an appropriate learning theory. An example of the new e-learning that occurs in a Maple immersion mode is useful at this point. Students are required to construct the slicing diagram of the domain of double integrals — both in assignments and the examination (see the first panel in the Slicing Diagram, Figure 1). This facilitates visualization and the correct set-up of double integrals and enables plotting of surfaces above an irregular domain [5, 6]. The other panels of Figure 1 provide three frames of an animation to illustrate how the 2D domain of integration is generated by adding the slices.

Slicing Diagram

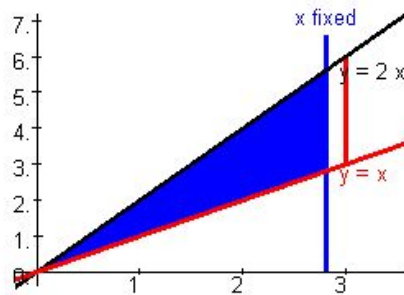
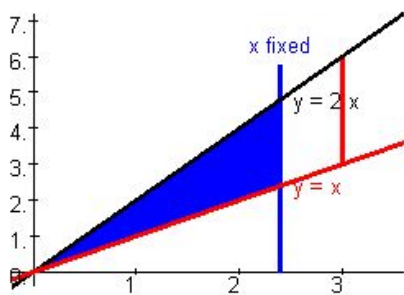
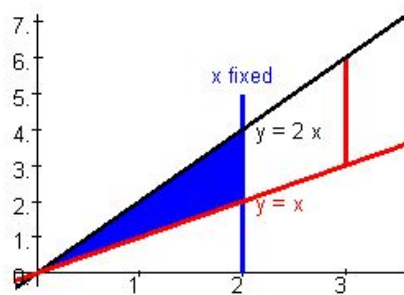
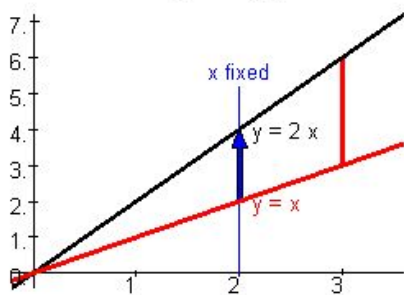


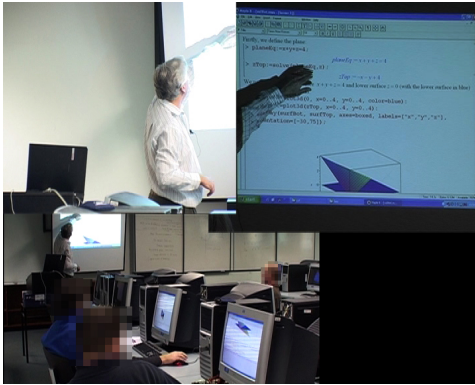
FIGURE 1: The slicing diagram of the domain for a double integral.

3 Videotaping and analysis

In recent years video has become a major tool in the study of school classrooms. Of all the data sources currently available to researchers in education, video data seems the most amenable to multiple analyses and to hold the greatest promise of significant research outcomes. At the University of Melbourne, David Clarke is the Director of the International Centre for Classroom Research (ICCR). The ICCR is a state of the art facility for the collection, storage and analysis of data, particularly video data, related to the study of learning and teaching.

In a previous study into pedagogical use of CAS [15], research methods used were observations of classes, analysis of students' work, responses to specially designed test instruments, analysis of student journals, use of feedback questionnaires and structured interviews. The attention paid to evaluation was atypical: most of the literature in undergraduate mathematics education discusses curriculum or pedagogical development but has limited and unimaginative evaluation. Evaluation is usually absent, anecdotal or using student questionnaires. However, the video research, undertaken as part of the project discussed in the article, is much more sophisticated and innovative in the context of undergraduate mathematics.

A central aim of this project was to video a few classes to help clarify how to proceed in more extensive studies in the future. Data collection was undertaken during the second half of 2005. Three weeks of classes (they are all held in two hour blocks in a computer laboratory) of the Maple immersion mode course Vector Calculus Methods (for Geospatial Science) were videotaped. The data related to any one double hour lesson consists of two videotape sources (two cameras): during the first hour, the lecturer (Bill Blyth — giving the presentation via projection of the Maple file on the front screen) and the class (downloading and viewing the presentation Maple file on their individual computer monitor) are videotaped. During the second hour, one camera remained in the lab (to videotape the class undertaking



(a) The class.



(b) A student interview.

FIGURE 2: Videos of the class and of the video stimulated recall interview.

practice with the help of the lecturer) and the other camera was shifted to a separate room to videotape the student interviews. (In future studies we will also conduct a lecturer interview.)

Figure 2(a) illustrates one frame from post processed digital videotape (an mpeg file). There are three panels (and a dark space at the right hand bottom corner), obtained from only two cameras which had to be arranged differently than in the usual school classroom where there would be a teacher camera and a class camera. In our first videotaped class, we discovered that because of the difference in lighting levels, one camera cannot properly deal with the capture of the lecturer and the much brighter projection screen. Because of the computer laboratory layout, we could capture several students, their monitors and the lecturer with one camera (see the bottom panel of Figure 2(a)). The top left panel in the figure is a cropped copy of the bottom panel, adjusted for brightness: it captures the lecturer's performance well.

The top right panel of Figure 2(a) came from the second camera which was dedicated to capture the screen. This is preferred to a direct screen capture since the camera additionally records the lecturer's pointing on the

screen: both physically as shown in this figure and when using a laser pointer (at other times). Note that the teaching point here concerns the 3D plot of the tetrahedron under a given plane and in the first octant (preliminary to constructing the 3D slicing diagram for a triple integral throughout this volumetric region). One of objectives of our research is to determine how the availability of the full and interactive Maple file affects the students' learning. The video shows that the students are viewing the same interactive 3D image — a point followed up in the interviews.

A frame of an interview is provided in Figure 2(b). The interviewer was Aleksandra Labovic who played no role in the teaching or assessment of this course. Since the lecturer is one of the researchers, this arrangement was made to ensure that students could freely participate in the interview without fear of discrimination. The interview protocol was only slightly modified from that used with school students. The methodology was the same (a video stimulated interview) in that students were invited to make some general comments about the lesson and then given the video control and asked to forward to any parts of the lesson that they would like to comment on. Even in this small pilot study, the interviews were a very rich source of data.

Another modification made, to ensure the study met high ethical standards, was that the examination Maple files were randomly coded so that the lecturer was unaware of a student's identity when marking the exam. In the usual assessment process for examinations, the student's identity is known.

Immediately following data collection, the video data was digitized for transcription and coding within Studiocode (software for video analysis) for a more sophisticated analysis. Interview data was also transcribed. Documentary data including lecture, assignment and examination Maple files were also saved.

4 Conclusion

Internationally, the typical use of CAS in undergraduate mathematics education is to support the learning of otherwise traditional courses. At RMIT Mathematics, a few courses have been developed and run in a Maple “immersion” mode where Maple is the universe of discourse. This project has commenced the exploration of the new e-learning occurring with this Maple immersion mode of teaching, learning and assessment.

The state of the art videotaping approach introduced here has demonstrated that the methodology established in school settings can be successfully modified to research the teaching and learning of technology rich undergraduate courses. In further work, we will undertake the analysis of our current data using Studiocode (specialist video analysis software) and expand the scope of our study from the current pilot project. Given the exploratory nature of this project, it was not appropriate to construct a rigorous evaluation design, but later work will include an evaluative component and this would be informed by the study reported in this paper.

The nature of the data (multi-camera video, post-lesson interviews, student and teacher material, questionnaires and assessment/examination Maple files) is such that it will be highly amenable to (future) multiple analyses. Such multiple analyses maximise the value to the research community of data that were initially funded to address one specific research question (local or international), now answered, but which are sufficiently complex to sustain multiple further analyses. Such secondary analyses are quite common for large databases such as TIMSS [2]. Further analyses in the future may require additional ethics approval: this depends on the specification of the current ethics approval. In this pilot study, the ethics approval application was closely related to the present study. However, some large international studies obtain ethics approval for analysis of the data by the members of the international research team from a variety of perspectives. This gives the research team a *carte blanche* to analyse the data from many and any per-

spectives, since the approved basic goal is “to understand the practices and meanings that characterise student and teacher behaviour in the classrooms being studied.”

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