Development of enquiry-oriented learning in the mathematical sciences

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Abstract

Arguably the largest challenge facing undergraduate students majoring in quantitative disciplines is the large gulf between high school mathematics and those skills eventually required in the workplace. The differences are not just in the level and depth of the disciplinary knowledge required but, more challengingly, in the types of learning and problem-solving methods employed. Here, I discuss recent developments in the curriculum at the University of Technology Sydney for undergraduates majoring in the Mathematical Sciences and related disciplines. In particular, I focus on the implementation of mathematical modelling workshops for students from their very first semester.

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1 Introduction

In 2015, a widespread review and renewal of the University of Technology Sydney (UTS) undergraduate subject offerings in Mathematical Sciences began. The primary aim of this process was to strengthen the curriculum offered and to ensure that subjects would be meeting the skills, both technical and professional, required by industry and academia in the coming years. Much of the motivation for this change arose from prior analysis of student performance and progression in the previously-offered subjects which highlighted potential points in the degrees where there was scope for improvement [1]. Additionally, more universally-noted concerns were raised regarding recent trends in student study techniques [2], in light of which some alternative assessments were deemed to be potentially beneficial. Furthermore, any changes to the subject offerings were required to deliver on the university-wide commitment to the UTS Model of Learning [3]. This provides a framework for a modern, research-inspired and practice-oriented education to prepare students for their lives and careers beyond UTS. Rather than solely maintaining the traditional focus on a degree's disciplinary knowledge, the model broadens this to develop within students an enquiry-oriented approach and an engagement with the needs of society.

The greatest perceived barrier to delivering on these promised outcomes was the wide gulf between learning strategies of many incoming students and those which are required in the workplace or in academic research. Much has been written about the rise of high school "coaching" services and the prioritising of rote learning procedures, often at the expense of developing mathematical enquiry and deeper understanding. Many students' primary learning strategies for high school mathematics are founded in pattern recognition techniques [4, 5]. That is, they can excel when presented with questions which are very similar to those for which they have already seen solutions [6]. When faced with unseen tasks, or problems phrased in a non-familiar fashion (such as in everyday, real-world language rather than already mathematically formulated) they fare less well. Common initial responses are "I can't remember the formula for this" or "I haven't seen this type of question in high school", in conjunction with an inability or unwillingness to progress. This is true even when the task might be relatively simple. In many cases, the students do not lack the ability to answer such questions, but have not yet developed either the confidence to explore new topics or the learning styles which might reward novel enquiry with new insights and abilities [7]. Furthermore, many students "strategise out" topics which they perceive to be more difficult when preparing for exams. That is, for exams with, say, a simple 50% pass mark, students might choose to study only what they believe to be the material on which the easiest marks might be obtained. Often the result of this is gaps in their skillset when progressing into future subjects. In this context, the need for a revamped curriculum, and the novel learning and assessment strategies required to implement it successfully, was seen to be vital to ensuring the success and competitiveness of the university's future graduates.

2 Discussion

The curriculum renewal for undergraduates majoring in the Mathematical Sciences centred initially on the core, compulsory first year, first semester subject in linear algebra and calculus. This previously had a pass rate below the benchmarks set by the UTS Faculty of Science and was seen as being especially affected by the issues of rote learning, student preparedness and exam strategising. As a prerequisite subject for later subjects along two different prerequisite paths (one in algebra and one in calculus/algebra) these issues were especially problematic. With a single pass mark for the subject, a student could, theoretically at least, score 100% on linear algebra and 0% on calculus and still score 50% and hence pass the subject and progress onto more advanced calculus subjects.

In the newer offering of the subject, Mastery Learning [8] was introduced as the primary assessment strategy for the subject. Table 1 summarises the new assessment structure and compares it to the old structure. Under the new framework, the subject is assessed by several shorter tests each with a high pass mark (in this case, 80%). To complete the subject successfully, a student must pass each of these tests. With such a high benchmark for each test, students are given multiple attempts (in this case, up to three) and the best mark of these attempts is counted. Because of the additional workload from writing and marking multiple tests, these assessments are run on an online platform. A large bank of questions was developed and a random selection from these is given to each student. Feedback and marking is instant upon submitting the solutions to the test. Additionally, students are given access to a similar bank of questions to self-pace and self-direct their own revision and practice as required; however, the Mastery tests are attempted under traditional invigilated closed-book conditions. This, in conjunction with other online resources [9], allows students who have entered university with a lower level of high school mathematics to bridge some gaps in their understanding and experience without forcing students who already have some of those skills to attempt as many practice questions.

In introducing Mastery Learning to the subject, and in light of the previously-identified issues with first year transition, it was feared, that there was a danger of encouraging student to believe that disciplinary knowledge alone was the be-all and end-all of the subject and that training to solve the already-formulated online test questions would be the best learning strategy. This, of course, is

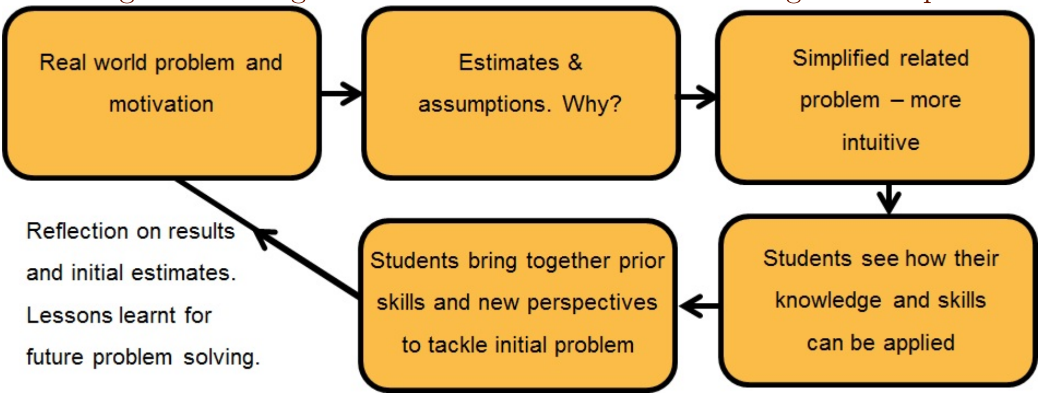
Table 1: Summary of the assessment structure before and after the implementation of Mastery Learning, including assessment weight.

Old Format, pre-Mastery Learning	**New Format, including Mastery Learning**
Mid-semester Test (closed book, pen-and-paper) 15%	Mastery Tests (closed book, online assessment, instant feedback) 54% (4%, 25% and 25%)
Tutorials (open book, pen-and-paper) 10%	
Computer labs 10%	Practical modelling workshops (group work, problem-based) 10%
Final Exam (3 hrs) (closed book, pen-and-paper) 65%	Final Exam (2 hrs) (closed book, pen-and-paper) 36%
Pass Criterion: at least 50% overall.	Pass Criterion: at least 80% of the marks on each of the three Mastery Tests plus satisfactory completion of at least seven of the ten workshops.

the antithesis of what the UTS Model of Learning seeks to develop in graduates. In response to this, and to deliver on the development of enquiry-oriented learning practices, the traditional pen-and-paper tutorials were replaced with group-based, practical workshops in mathematical modelling and applications. Although more simplified and structured than problems arising in current research or industrial practice, the workshops provided an excellent first step away from some of the learning styles of high school mathematics and towards those of a hands-on, practical applied mathematician.

The workshops typically open with a real-world industry-inspired problem whose solution cannot be obtained by any calculation which students will have seen before. Students are then encouraged to think about the context and make reasoned approximations and assumptions to estimate which ranges

Figure 1: The general framework for each modelling workshop.



of solutions might be plausible. The workshop leader can then present similar, but simplified cases of a closely related system and explain which mathematical procedures are required and, more importantly, why. Finally, students are then able to obtain a solution to the initial problem and reflect on why/whether their initial intuition was reasonable or accurate.

The workshop structure is designed to build the confidence of students to attempt problems whose type they have perhaps not seen before, a skill often underdeveloped by heavily "coached" students. Furthermore, being able to make reasoned and justifiable quantitative estimates for complex problems (even if only as a sense-check) mimics best practice in many industries and in research.

The format, which promotes discussion of mathematics in context, arguably makes larger class sizes a benefit and not a hindrance to the educational experience and quality delivered. Whereas classes of up to 45 undergraduates are problematic for traditional pen-and-paper tutorials, nine groups of five students can provide robust discussion and a diversity of viewpoints and assumptions. Figure 1 illustrates the conceptual framework around which each of the workshops was developed.

It was decided that workshops should not be heavily weighted in the assessment of the subject, so that the main factor deciding whether or not a student passes or failes the subject would be his/her performance on the Mastery tests which assess the core disciplinary knowledge. Nonetheless, to ensure that the workshops were still valued, each of the ten workshops were worth 1% of the final mark but with a minimum completion requirement of 7 out of 10. That is, unless a student satisfactorily participated in a minimum of seven workshops, he/she would not be able to complete the subject. This was clearly stated at the beginning of the subject and the reasons for this were explained to students. The 1% assessment weight was not awarded necessarily for providing all the correct answers to the workshop problems, rather for participating and providing reasoned and justified estimates and approximations when asked.

One of the most successful and widely enjoyed workshops was based on the popular board game Monopoly. Students were presented with an image of the board and a brief reminder of the basic rules and asked the simple (but ill-defined) question “which square is best to own?” Very quickly, several students queried the ambiguity of the idea of “best”, since the most costly for an opponent to land upon were both the most costly to purchase and not necessarily that likely to be visited. Across the whole class, it was agreed to interpret the question as “which purchasable square is a player most likely to land on?” Calculating this is anything but a trivial task and students realised that they would not be able to obtain the answer.

However, many students were able to think more broadly about the problem and obtain the correct answer. They reasoned that an average player spends much of the game in the Jail square and that, rolling two regular fair six-sided dice, a typical move is around six, seven or eight squares so the orange properties (which lie this many squares after the Jail square) would likely be frequently visited. Other groups who did not reach this conclusion nonetheless provided some very reasonable answers, supported by sensible justifications. For example, thinking that the squares six, seven or eight squares ahead of Go might be the most likely was a common answer, as were any of the squares

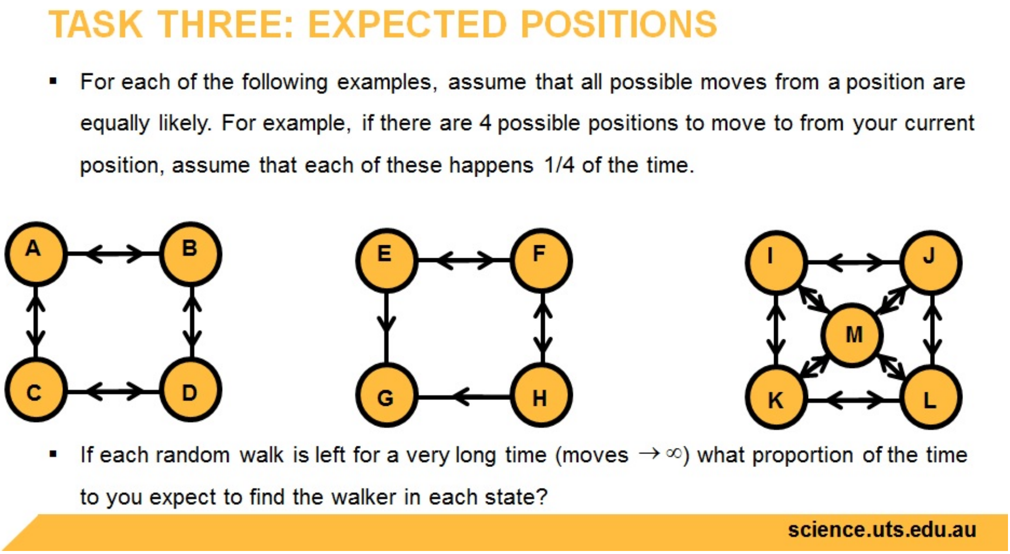


Figure 2: Screenshot of one of the tasks from the Monopoly-themed workshop.

for which there is an "Advance to ..." card in the game. As groups were not assessed on getting the question right, all such justified estimates were encouraged and discussed. (Whereas the orange squares are the most likely visited colour set, the single purchasable square most likely to be visited is Trafalgar Square, as it is both close enough to the Jail to still be influenced and also has an "Advance to Trafalgar Square" card favouring it.)

Once groups produced their initial estimates, it was explained that such a problem could be represented mathematically as a random walk on a graph. After a brief explanation of directed graphs, they were asked to analyse random walks on the three graphs presented in Figure 2.

Most groups rapidly realised that the ABCD graph was completely symmetric and hence, in the long run, a random walker would spend approximately a quarter of the time in each. For the EFGH graph, most groups realised that G was absorbing and hence, eventually, it would become certain that a player would be stuck in G. Analysis of the IJKLM graph was much harder, but it was observed that I, J, K and L should be equally likely and that M should be a little more likely to be visited, owing to it being better connected.

The workshop then explained the idea of transition matrices for a random walk and showed that steady-state distributions could be found through calculation of appropriate eigenvectors. This was a topic which all students would have seen before, but certainly not in the context of a board game or a random walk.

Finally, the workshop returned to the initial Monopoly problem. It was realised that this problem would require a 41 node graph (40 squares, but a player can be In Jail or Just Visiting Jail) and hence a 41 x 41 transition matrix and a 41 dimensional eigenvector. While this task was far beyond the scope of the workshops, students were able to appreciate that they did have both the understanding and the mathematical tools (if not the time or the inclination) to solve this seemingly complicated task. The workshop concluded by bringing up the results from an external website which had indeed done this calculation and groups were able to reflect on how their initial estimates compared to the calculated solution.

In 2015, the ten workshops presented covered diverse topics including:

- mortgage calculation and compound interest;
- telecommunications satellites;
- data science and regression models;
- combinatorics (and how many ways to navigate the Melbourne CBD grid system);
- projectiles, kinetics and sports science;
- cryptography and data security;
- graph theory (and how to win at Monopoly);

- game theory (in economics and ecology);
- data science and statistical estimation;
- population and conservation biology.

3 Results and conclusions

Reponses to the workshops were almost universally positive. Internally, student satisfaction is measured based on the Student Feedback Survey (SFS) with mean ranks recorded on a five point scale with 5.0/5 meaning all students gave the most positive response and 1.0/5 meaning all students gave the most negative. In implementing the new structure, with both Mastery Learning and the enquiry-based workshops, SFS were higher than in previous semesters, with no single category returning a score of below 4.2/5. By comparison, in the final semester of its predecessor, the subject did not return a single score on the questionnaire higher than 4.09/5 and had one score as low as 3.55/5. Furthermore, the written responses within the SFS were also largely positive, including:

"I enjoyed the workshops as they allowed everybody to participate in the learning. They also provided a chance to work with Mr Woodcock in small groups, which seemed to be a good working environment".

"This tutorial session was the highlight of my week! The different areas of learning were incredibly interesting and I learned a lot. My favourite session would have to be learning about false credit cards and modular. Stephen was a very good teacher in the tutorials."

However, this push towards enquiry-oriented learning practices was not universally liked by students. On the SFS, as anticipated, a minority of students complained that the workshops did not focus solely on training them for the format and questions of their exam assessments and hence were deemed to be a poor use of their time. Despite an explanation that the workshops were not primarily focussed on disciplinary knowledge, but rather other more general mathematical "soft skills", several students still expressed a preference for more high school-like procedural, rather than context-based mathematics.

"The tutorial is so pointless. It should be used to help us pass the mastery tests by going through practice questions in class with the tutor. Overall I am disappointed in the tutorials."

"The tutorial was a complete WASTE of time! honestly I think the tutorial didn't help any student at all. It was irrelevant material that had nothing to do with the exam material and was just for team work."

Overall, though, the initial implementation of this learning and assessment scheme was deemed a success, although the possible benefits will not be fully known until the current cohort of undergraduates reaches upper-level subject and, eventually, the workplace. It is our contention that opening up students to more open-ended thinking and practice-based problems sooner will only be beneficial, in terms of mathematical confidence, ability and outlook. Whether or not this proves to be the case remains to be seen.

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