On improving the accuracy of simple aerial towed-cable system models

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Abstract

We outline a novel procedure for improving the accuracy of a simple one-link aerial towed-cable system model. The primary objective of this work is to provide a systematic framework for matching the dynamical motion of a simple aerial towed-cable system model with that of a correspondingly more complex model. The final outcome is to achieve a compromise between a model's representativeness and the ease with which it can be used for control purposes. By modifying the cable length and the payload drag coefficient of the simple model, the equilibrium position of the cable tip may be matched analytically to that of the more complex model. The modified cable length and drag coefficient are then used in dynamic simulations and shown to dramatically improve the accuracy of the simple model.

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Contents

1 Introduction

Highly complex and accurate mathematical models of aerial towed-cable systems, capable of capturing all the significant dynamical motion of the physical system, have been extensively used by researchers with encouraging results [1, 2, 3, 4, e.g.]. More recently the control of such system attracted interest [2, 3, 4]. Although sophisticated models can accurately simulate and predict the motion of the physical system, their inherent complexity renders them cumbersome to use for control system design purposes, particularly when trajectory optimisation techniques are sought. Simple models tend to be more widely used in control system design, but unless verified, their use can result in non-representative simulations and/or inept controllers. The question is: Is it possible to reconcile the competing demands of model simplicity and representativeness, in a systematic and relatively straightforward manner? Hence the main objective of this work is to improve the accuracy of a simple model of an aerial towed-cable system model using a more complex model as a reference. The dynamic variable of interest here is the position of the cable tip, since knowledge of this variable is central to the develop-

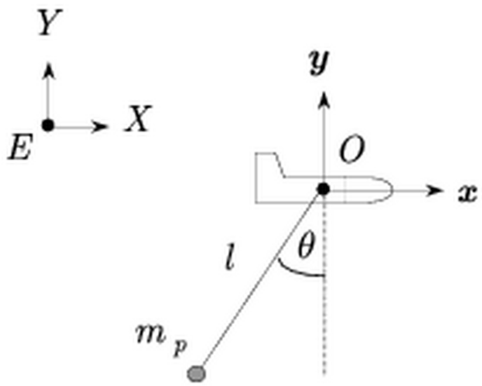

Figure 1: Simplified aerial tether model.

ment and successful performance of controllers for aerial towed-cable systems. For simplicity, only two-dimensional in-plane motion will be considered, the aircraft is assumed to remain at constant altitude, the payload is assumed spherical, and the cable length is fixed.

2 Aerial towed-cable system modelling

In this section, two models of the aerial towed-cable system possessing varying degrees of sophistication are presented. These models are classified as either being single-link or multi-link, the simplest of which being the single-link variety depicted in Figure 1. This model is the simplest possible representation of the system. It represents a compromise between complexity and ease with which it can be used for control system design purposes. It is assumed that the aircraft, whose motion (horizontal) is prescribed for all time, has an infinite mass so its motion remains unaffected by the dynamics of the cable/payload. The cable is assumed to be rigid, have a constant circular diameter and to be straight at all times. The length of the cable varies by prescribing the reel acceleration of the cable. The cable is attached to a spherical payload at the payload's centre of gravity. The dominant external forces acting on the aerial towed-cable system are aerodynamic and gravitational, lumped at the payload's centre of gravity. Only simple velocity-dependent aerodynamic drag acting on the payload will be considered, with the aerodynamic force acting on the cable ignored in this work for the single link model. The payload is assumed to have a constant drag coefficient.

The position of the towed-body is described relative to the aircraft. The axes Oxy are attached to the aircraft center of mass and translate with it relative to the Earth-fixed coordinate system EXY. By employing Lagrange's Equations with the in-plane cable angle θ and the length l used as generalized coordinates (see Figure 1), the equations of motion for the system are:

$$\dot{q}_1 = \dot{x}\,, \tag{1}$$

$$\dot{q}_2 = \ddot{x} = u_1\,, \tag{2}$$

$$\dot{q}_3 = \dot{\theta}\,, \tag{3}$$

$$\begin{aligned}\dot{q}_4 &= \frac{1}{l}\left(-2\dot{l}\dot{\theta} + \ddot{x}\cos\theta - g\sin\theta\right) - \frac{\pi\rho_{\text{air}}{d_p}^2C_d}{l\left(8m_p + \pi\rho_c l{d_c}^2\right)}\left(l\dot{\theta} - \dot{x}\cos\theta\right) \\ &\quad\times\sqrt{\left(\dot{x} - \dot{l}\sin\theta - l\dot{\theta}\cos\theta\right)^2 + \left(-\dot{l}\cos\theta + l\dot{\theta}\sin\theta\right)^2}\,,\end{aligned} \tag{4}$$

$$\dot{q}_5 = \dot{l}\,, \tag{5}$$

$$\dot{q}_6 = \ddot{l} = u_2\,, \tag{6}$$

where x is a kinematic constraint representing the horizontal range of the aircraft. The mass density and diameter of the cable are ρ_c and d_c, respectively, whereas m_p, d_p, C_d are the payload mass, diameter and drag coefficient, respectively. The air density and gravitational constant are ρ_{air} and g, respectively. The actuators (u_1,u_2) used to control the dynamics of the system are the forward acceleration of the aircraft $\ddot{x}$ and the cable reel acceleration $\ddot{l}$.

Due to its inherently simple nature, the equilibrium configuration of the towed-body can be found analytically by setting all time derivatives to zero in Equations (1)–(6). It can be shown that the equilibrium cable angle θ_e

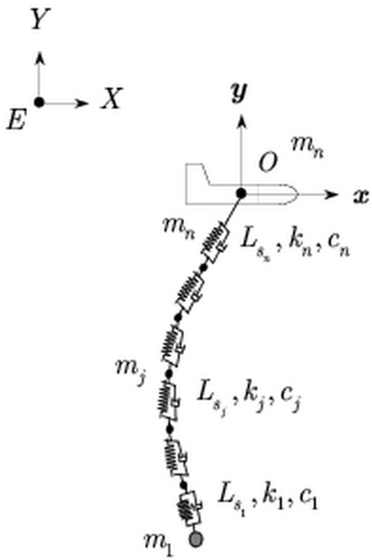

Figure 2: Flexible multi-link aerial tether model.

and the position (x_e, y_e) of the cable tip are

$$\theta_e = \arctan\left(\frac{\pi\rho_{\text{air}}{U_E}^2 C_d {d_p}^2}{g\left(8m_p + \pi\rho_c l_e {d_c}^2\right)}\right), \quad (7)$$

$$l_e = \sqrt{{x_e}^2 + {y_e}^2}, \quad (8)$$

where U_E is the steady state towing speed, l_e is the corresponding tether length, and (x_e, y_e) are the coordinates of the tether tip relative to the aircraft.

With respect to the more representative multi-link mathematical models of the system, these models fall into two main classes, each differentiated by their treatment of the longitudinal elasticity of the cable and coordinate systems used to describe the motion of the system. Figure 2 shows the aerial towed-cable system physically discretised using flexible links. This is the complex aerial towed-cable system model which is treated as the "truth" model in this paper. Williams et al. [4] gave the full treatment of the equations of motion for the flexible multi-link system. Essentially, this model employs a lumped parameter representation for the cable, whereby the cable is physically discretised using a sequence of n point masses interconnected via viscoelastic springs of unstrained length L_{s_j}, stiffness k_j, and damping constant c_j. The corresponding element mass is concentrated to a single point, represented by m_j. In a similar manner to the simple model, the cable elements use coordinates that are relative to the aircraft. The cable is assumed to be perfectly flexible in the span-wise case and hence the point masses are assumed to act as frictionless hinges.

3 Methodology

The fundamental idea of this work is to improve the predictions of the single-link model. The key differences between the simple model and the "true" system are: 1) tether drag, 2) tether mass, and 3) tether flexibility. The effects of tether drag and mass depend on the instantaneous angle of the tether to the vertical and impact the variation of the swing dynamics, whereas tether flexibility alters the range of the payload from the aircraft due to the geometric shortening of the tether from curvature combined with variations in longitudinal stretch of the tether. To improve the simple model, the drag coefficient of the payload and the tether length are adjusted:

$$C_{d_{\text{new}}} = C_d\,(1 + p_1)\ , \tag{9}$$

$$l_{\text{new}} = l\,(1 + p_2)\ , \tag{10}$$

where p_1 and p_2 are the adjustment parameters to be determined. In general, these parameters depend dynamically on the cable motion. Assuming that the position of the payload (x_E, y_E) is known from the multi-link model, as shown in Figure 3, then

$$p_2 = \left(\frac{l_E}{l}\right) - 1\,. \tag{11}$$

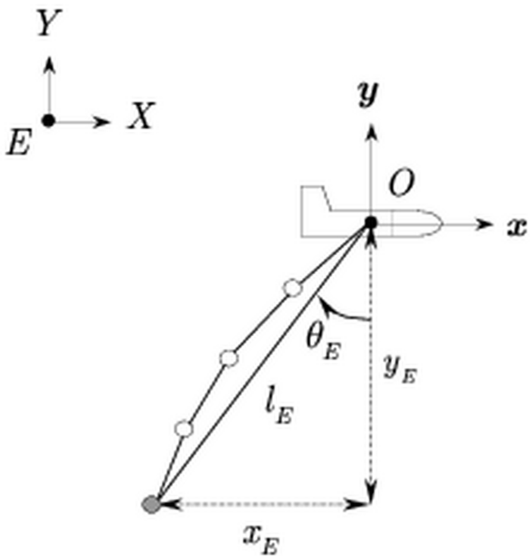

Figure 3: "Equivalent" equilibrium configuration of the multi-link model.

Using Equation (4),

$$p_1 = \frac{1}{\mathcal{A} l_E}\left(-2\dot{l}_E\dot{\theta}_E + \ddot{x}\cos\theta_E - g\sin\theta_E\right) - \frac{\ddot{\theta}_E}{\mathcal{A}} - 1\,, \tag{12}$$

where

$$\mathcal{A} = \frac{\pi\rho_{\text{air}} d_p^2 C_d}{l_E\left(8m_p + \pi\rho_c l_E d_c^2\right)}\left(l_E\dot{\theta}_E - \dot{x}\cos\theta_E\right) \times \sqrt{\left(\dot{x} - \dot{l}_E\sin\theta_E - l_E\dot{\theta}_E\cos\theta_E\right)^2 + \left(-\dot{l}_E\cos\theta_E + l_E\dot{\theta}_E\sin\theta_E\right)^2}, \tag{13}$$

$$\tan\theta_E = \frac{x_E}{y_E}\,, \tag{14}$$

$$l_E = \sqrt{{x_E}^2 + {y_E}^2}\,, \tag{15}$$

The time derivatives of θ_E are obtained by differentiating Equation (14):

$$\dot{\theta}_E = \frac{\left(\frac{\dot{x}_E}{y_E} - \frac{x_E \dot{y}_E}{{y_E}^2}\right)}{1 + \tan^2\theta_E}\,, \tag{16}$$

$$\ddot{\theta}_E = \frac{\left(\frac{\ddot{x}_E}{y_E} - 2\frac{\dot{x}_E \dot{y}_E}{{y_E}^2} + 2\frac{x_E \dot{y}_E^2}{{y_E}^3} - \frac{x_E \ddot{y}_E}{{y_E}^2}\right)}{(1 + \tan^2\theta_E)} - 2\dot{\theta}^2 \tan\theta_E\,. \tag{17}$$

The "equivalent" radial velocity

$$\dot{l}_E = \frac{(\dot{x}_E x_E + \dot{y}_E y_E)}{l_E}\,. \tag{18}$$

Equation (12) provides the exact value that p_1 must take in order for the simple model to match the multi-link model. However, it relies on the actual values of θ_E, $\dot{\theta}_E$, $\ddot{\theta}_E$, l_E, and $\dot{l}_E$. For control design, these are unknown and hence Equation (12) cannot be directly applied. Instead, for motions that occur around the cable equilibrium angle, we have that $\dot{\theta} \to 0$, so that $\ddot{\theta} \approx 0$. Furthermore, for the equilibrium position, Equation (12) evaluates to

$$p_{1_E} = \tan\theta_E \frac{g\left(8m_p + \pi\rho_c l_E {d_c}^2\right)}{\left(\pi\rho_{\text{air}} {U_E}^2 C_d {d_p}^2\right)} - 1\,. \tag{19}$$

Thus, we approximate p_1 using a series expansion in terms of $\dot{\theta}$ as

$$p_1 \approx p_{1_E} + f(l_E, \dot{\theta}, \dot{\theta}^2, \dots) \tag{20}$$

Neglecting second and higher order terms in f leads to the empirical approximation

$$p_1 \approx p_{1_E} + \left(\frac{l_E}{K_1 + \frac{K_2}{l_E}}\right)\dot{\theta}\,, \tag{21}$$

where K_1 and K_2 are constants that depend on cable elasticity. These coefficients are determined through trial and error in this work.

In summary, values of p_1 and p_2 can be obtained exactly from simulation results with the multi-link model using Equations (11) and (12). Such results are useful in understanding how p_1 and p_2 are influenced by the cable dynamics. However, for improved a priori predictions using the simple model, Equation (21) is used to simulate the payload motion around the nominal equilibrium position.

4 Case study

The performance of the improved simple model is demonstrated through a dynamic simulation example. The simulation demonstrates how the refined simple model compares to the flexible multi-link model under the action of applied control at the aircraft. The control applied to each of the models is represented by Figure 4. The parameters governing the case study simulation are given in Table 1. The simulation begins with the models in their respective equilibrium configurations and progresses under the action of the applied control until final equilibrium is achieved.

The results of the simulation are given by Figure 5 through to Figure 7, which show the trajectory the cable tip follows during the simulation along with the value of the parameter p_1 over the simulation. The value of the parameter p_2 was a constant value of -0.0451 over the simulation since no cable reel acceleration was applied. The effectiveness and performance of the matching procedure is exemplified by these plots. No adjustment to the parameters of the simple model leads to large errors in the payload position, whereas the improved model shows much closer correspondence to the multi-link model. Note that the plots show that when the aircraft speed increases the payload moves upwards and further behind the aircraft due to the increased drag. The maximum error in the x-coordinate and y-coordinate of cable tip during the simulation was 39.77 m and 41.03 m respectively, equivalent to 2.65% and 2.74% when normalized with respect to the cable length.

Table 1: Parameters governing case study simulation.

Parameter	Value
Payload mass [kg]	200
Cable diameter [mm]	2
Payload area [m^2]	0.75
Cable tangential drag constant	0.022
Payload drag constant	0.47
Cable normal drag constant	1.1
Initial tow speed [m/s]	45
Cable Elasticity [N/m^2]	1.2×10^{11}
Cable length [m]	1500
Cable Damping	100
Cable density [kg/m^3]	3000
Number of cable elements	9
Air density [kg/m^3]	1.225
Gravity constant [m/s^2]	9.81
K_1	29
K_2	3×10^4

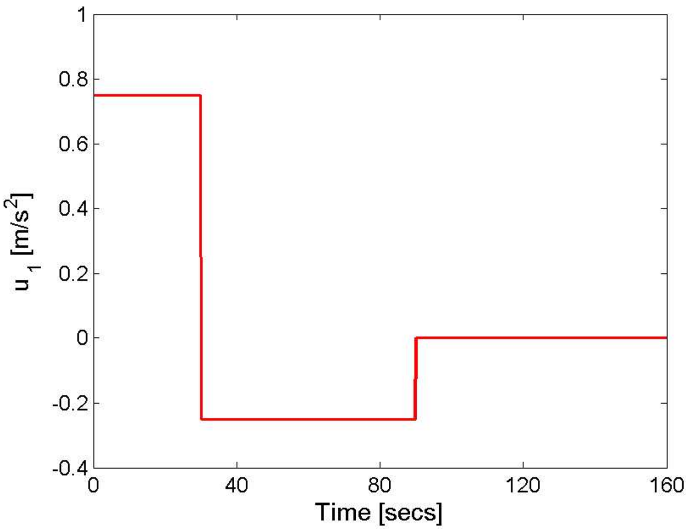

Figure 4: Control actuation during simulation- aircraft acceleration $\ddot{x}$.

5 Conclusions

An improved simple model for an aerial-towed cable system was developed by prescribing changes to the cable length and the payload drag coefficient of the simple model. Exact matches to a complex cable model are possible using the simple model provided that simulation data is available from the complex model. For prediction purposes, a series expansion of the cable motion around the equilibrium position combined with an empirical approximation for the pendulum damping term was found to give accurate results.

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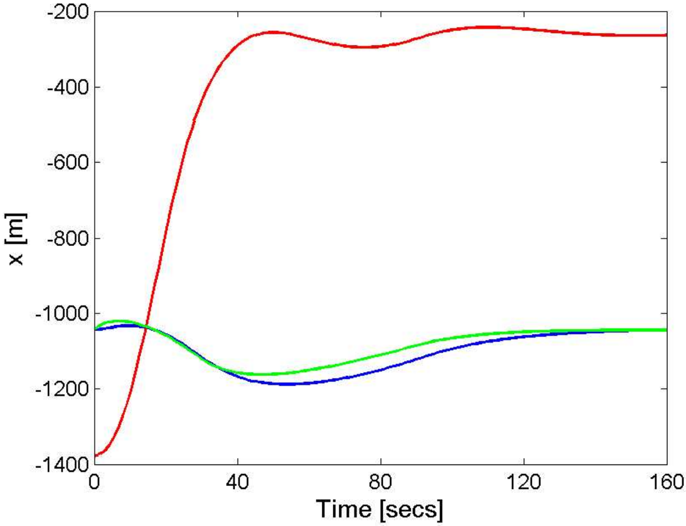

Figure 5: Simulation results, x-coordinate of cable tip: (red) simple model, (blue) refined single-link model, (green) flexible multi-link model.

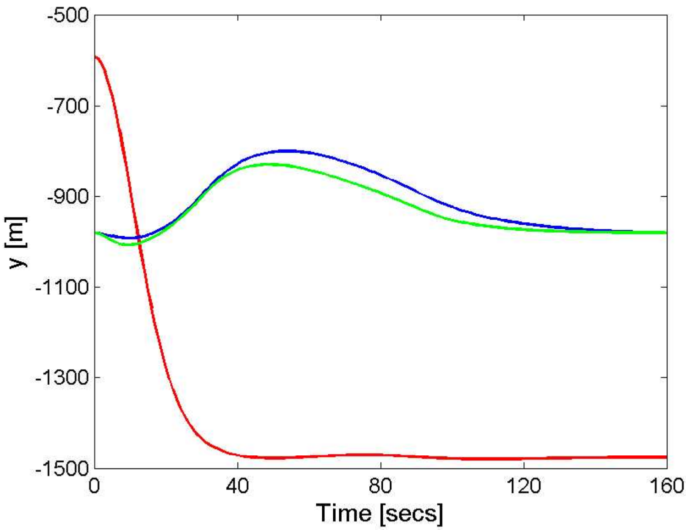

FIGURE 6: Simulation results, y-coordinate of cable tip: (red) simple model, (blue) refined single-link model, (green) flexible multi-link model.

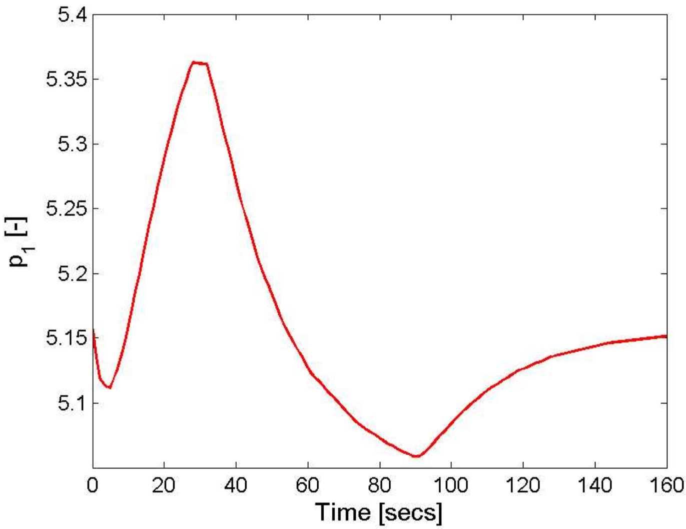

Figure 7: Simulation results: p_1 adjustment for simple model.

References

[1] C. K. H. Chin, R. L. May and H. J. Connell. A numerical model of a towed cable-body system. *ANZIAM J.*, 42(E):C362–C384, 2000. [Online] http://anziamj.austms.org.au/V42/CTAC99/Chin. C556

[2] P. Trivailo, D. Sgarioto and C. Blanksby. Optimal control of aerial tethers for payload rendezvous. *Proceedings of the 5th Asian Control Conference*, Melbourne, Australia, July 20–23, 2004, pages 285–294. C556

[3] P. Williams, D. Sgarioto and P. Trivailo. Optimal control of an aircraft-towed flexible cable system. *Proceedings of the 11th Australian International Aerospace Congress*, Melbourne, Australia, 13–17 March, 2005, pages 1–21. C556

[4] P. Williams, D. Sgarioto and P. Trivailo. Motion planning for an aerial-towed cable system. *AIAA Guidance, Navigation, and Control Conference and Exhibit*, San Francisco, California, Aug. 15–18, 2005, pages 1–31. C556, C560