

Surf-zone kinematics

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Abstract

A new method is described for making field measurements of broken surf fronts (turbulent bores) on coastal beaches. The position of a Garmin watch attached to a bodyboarder riding the surf front is recorded every second. The results of a large number of experiments on various beaches indicate that the surf fronts are uniformly retarded. From this, the retardations and initial breaking wave speeds are then obtained. The constant value of the retardation may vary from one wave to the next. The results are applied to derive a heuristic method for determining the approximate width of the surf zone by noting the total time of travel of any broken surf front.

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1 Introduction

The surf zone is that region of an ocean or large lake where waves break near the shoreline and then travel towards the water's edge. This may occur at a beach, a rocky shore or a headland. Many people use the surf that runs onto beaches for recreation purposes and, in some cases, for competitive events.

In 1996, 2010 and 2012, three separate accidents occurred in heavy seas during events at the Australian Surf Lifesaving Championships. In each case a young competitor drowned. Three days after the third tragedy, we started to develop a quantitative Surf Hazard Rating to help officials to decide when to postpone the competition or move it to a less dangerous venue.

For our Surf Hazard Rating (SHR) model [2, 4], we identified seven major characteristics within the surf zone that give rise to various degrees of hazard. These include wave height, wave type, wave period, surf zone width, surface turbulence, longshore current (sweep) and offshore current (rip). In the majority of surfing situations, the two dominant characteristics are wave height and surf zone width. The former is measured approximately using the height of a person riding a craft when at the base of a breaking wave. The latter is the distance covered by a broken surf front from the initial point of wave breaking to the point where the surf front ceases to exist. This zone

width is more difficult to measure, so we carried out experiments in the surf zone to see if there were kinematic relations that might be useful.

A great amount of research on surf-zone behaviour has been reported in the past [1, 3], but it is mainly based on laboratory experiments and mathematical modelling of the perceived fluid dynamical equations. Field experiments have been limited to small surf with measuring instruments fixed in place within the surf or video recordings with the inherent aberration effects.

At this stage little is known about the total macroscopic behaviour of the water in the surf zone. It is both turbulent and laminar underneath the surface, this being a moveable interface (waves and bores) between the air and the water caused mainly by distant and local weather conditions and the local bathymetry. Eventually a wave approaching the shore may break forming a surf front of aerated turbulent water/air mixture which travels shorewards within the surf zone. This article sheds more light on the understanding of the kinematics of these broken surf fronts, known colloquially as ‘breakers’.

2 Experiments

A waterproof Garmin Forerunner 910XT watch, using satellite navigation to record its position every second, was attached to the wrist of a bodyboard rider. The rider then rode waves from their breaking point directly towards the shore with the watch arm held steady on the bodyboard just clear of the turbulent surf front.

A total of seventy-four experimental rides were conducted over eleven days at six separate beaches on the Gold Coast, Queensland in 2014 and 2015. Wave heights at breaking varied from small (< 1.0 m) through medium (1.0 m to 1.9 m) to large (2.0 m or more). These experiments represent a completely new approach to obtaining kinematic data about moving surf fronts. The authors are both experienced applied mathematicians and body surfers. The lateral distances covered for the experimental rides ranged

from 12.7 m to 136.5 m, while the ride times were between 5 and 28 seconds. The watch ran continuously during each experimental period of approximately one hour. Much of the time was spent travelling out through the surf zone and waiting for a reasonably-sized wave. The watch recorded the time and position at each one-second interval. From this data, a spreadsheet was produced for the distance travelled during each second of a ride.

The start and time interval of each ride was identified by an observer on the shore with a stopwatch synchronised to the Garmin watch. The observer also noted the wave height from the position of the wave crest at break point compared with the height of the bodyboarder on the wave face. A further identification of the start of each ride on the spreadsheet was provided by the greatest distance moved towards the shore in any one second interval. The end of each ride occurred when the distance moved in one second fell below 2 m, for then the bodyboarder was no longer able to ride the wave and the surf front was about to disappear.

3 Results

Figure 1 shows the intermediate distances (d) in metres covered during the experimental ride of the first 1.7 metre wave in **Table 2** plotted against the progressive time (t) in seconds.

Tables 1, 2 and 3 show the total ride times and total distances travelled for three groups of turbulent bores (surf fronts) classified by small, medium and larger wave heights at breaking. **Table 1** is for small surf with initial wave heights less than 1.0 m. **Table 2** is for medium surf with wave heights from 1.0 m up to less than 2.0 m. **Table 3** is for larger surf generated by waves 2.0 m or higher. The most notable feature of the experiments is that every one of the seventy four rides had an $R^2 > 0.99$ for a quadratic fit of d versus t , and therefore the details are included in the final column of each table.

Table 1: Small surf fronts.

| Wave height (m) | Total time (s) | Total distance (m) | Quadratic fit |
|--------------------|----------------|-----------------------|---------------------------|
| 0.5 | 5 | 13.0 | $-0.12t^2 + 3.16t + 0.05$ |
| 0.5 | 7 | 18.3 | $-0.03t^2 + 2.86t - 0.02$ |
| 0.6 | 8 | 23.3 | $-0.11t^2 + 3.80t + 0.11$ |
| 0.7 | 6 | 19.7 | $-0.24t^2 + 4.71t + 0.03$ |
| 0.7 | 9 | 28.8 | $-0.14t^2 + 4.41t + 0.39$ |
| 0.7 | 12 | 39.2 | $-0.14t^2 + 4.87t + 0.65$ |
| 0.7 | 11 | 37.2 | $-0.05t^2 + 4.06t - 0.20$ |
| 0.7 | 8 | 26.8 | $-0.13t^2 + 4.39t + 0.12$ |
| 0.7 | 10 | 39.1 | $-0.11t^2 + 4.97t + 0.35$ |
| 0.8 | 11 | 34.6 | $-0.05t^2 + 3.68t + 0.17$ |
| 0.8 | 11 | 39.5 | $-0.07t^2 + 4.34t + 0.10$ |
| 0.8 | 9 | 29.5 | $-0.07t^2 + 3.85t + 0.34$ |
| 0.8 | 13 | 42.6 | $-0.09t^2 + 4.27t + 0.85$ |
| 0.8 | 9 | 31.0 | $-0.14t^2 + 4.67t + 0.29$ |
| 0.8 | 12 | 37.7 | $-0.08t^2 + 4.04t + 0.32$ |
| 0.8 | 6 | 20.1 | $-0.22t^2 + 4.57t + 0.31$ |
| 0.8 | 8 | 20.0 | $-0.08t^2 + 3.03t + 0.38$ |
| 0.8 | 5 | 12.7 | $-0.12t^2 + 3.11t + 0.02$ |
| 0.8 | 9 | 32.4 | $-0.17t^2 + 5.04t + 0.49$ |
| 0.8 | 5 | 13.7 | $-0.07t^2 + 3.10t - 0.06$ |
| 0.9 | 17 | 55.7 | $-0.04t^2 + 3.91t + 0.91$ |
| 0.9 | 20 | 67.9 | $-0.07t^2 + 4.67t + 2.47$ |
| 0.9 | 11 | 37.0 | $-0.06t^2 + 3.98t + 0.20$ |

Table 2: Medium surf fronts (continued on next page).

| Wave height (m) | Total time (s) | Total distance (m) | Quadratic fit |
|--------------------|----------------|-----------------------|---------------------------|
| 1.0 | 7 | 26.2 | $-0.21t^2 + 5.18t + 0.33$ |
| 1.0 | 8 | 26.3 | $-0.15t^2 + 4.46t + 0.15$ |
| 1.0 | 11 | 42.7 | $-0.05t^2 + 4.32t + 0.55$ |
| 1.0 | 20 | 66.2 | $-0.03t^2 + 3.93t + 1.02$ |
| 1.0 | 12 | 42.2 | $-0.10t^2 + 4.63t + 0.92$ |
| 1.1 | 12 | 58.7 | $-0.09t^2 + 5.96t + 0.01$ |
| 1.1 | 11 | 57.7 | $-0.13t^2 + 6.68t - 0.32$ |
| 1.2 | 5 | 29.4 | $-0.31t^2 + 7.44t - 0.07$ |
| 1.2 | 17 | 70.7 | $-0.14t^2 + 6.28t + 2.23$ |
| 1.2 | 18 | 64.8 | $-0.08t^2 + 5.00t + 1.30$ |
| 1.2 | 16 | 63.2 | $-0.09t^2 + 5.30t + 0.66$ |
| 1.3 | 20 | 109.2 | $-0.07t^2 + 6.76t + 2.38$ |
| 1.3 | 13 | 64.1 | $-0.04t^2 + 5.52t + 0.54$ |
| 1.3 | 13 | 48.4 | $-0.06t^2 + 4.55t - 0.55$ |
| 1.3 | 18 | 72.9 | $-0.07t^2 + 5.37t + 0.22$ |
| 1.3 | 11 | 41.0 | $-0.13t^2 + 5.05t + 0.66$ |
| 1.4 | 15 | 72.3 | $-0.09t^2 + 6.02t + 0.94$ |
| 1.4 | 13 | 56.3 | $-0.12t^2 + 5.90t + 0.26$ |
| 1.4 | 21 | 80.1 | $-0.09t^2 + 5.46t + 2.54$ |
| 1.4 | 16 | 62.4 | $-0.09t^2 + 5.23t + 0.89$ |
| 1.5 | 17 | 62.0 | $-0.09t^2 + 5.17t + 1.07$ |
| 1.5 | 13 | 51.2 | $-0.15t^2 + 5.91t - 0.30$ |
| 1.5 | 18 | 69.5 | $-0.08t^2 + 5.10t + 0.74$ |
| 1.5 | 10 | 43.9 | $-0.13t^2 + 5.63t + 0.81$ |
| 1.5 | 8 | 32.2 | $-0.13t^2 + 5.16t - 0.83$ |
| 1.5 | 9 | 31.7 | $-0.14t^2 + 3.74t - 0.04$ |
| 1.5 | 12 | 45.6 | $-0.12t^2 + 5.09t + 0.87$ |
| 1.5 | 12 | 40.6 | $-0.10t^2 + 4.61t + 0.38$ |

Figure 1: Typical distance-time relationship.

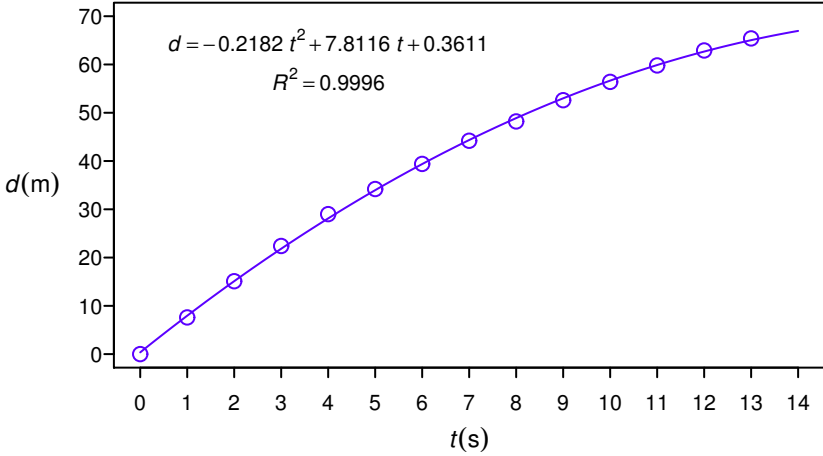


Table 2 continued: Medium surf fronts.

| Wave height (m) | Total time (s) | Total distance (m) | Quadratic fit |
|-----------------|----------------|--------------------|---------------------------|
| 1.6 | 19 | 76.7 | $-0.08t^2 + 5.45t + 2.18$ |
| 1.6 | 19 | 68.2 | $-0.08t^2 + 5.11t + 0.60$ |
| 1.6 | 19 | 82.0 | $-0.10t^2 + 6.21t + 1.43$ |
| 1.6 | 22 | 82.6 | $-0.09t^2 + 5.68t + 2.07$ |
| 1.6 | 12 | 44.4 | $-0.09t^2 + 4.67t + 0.60$ |
| 1.6 | 16 | 80.1 | $-0.10t^2 + 6.60t + 1.36$ |
| 1.6 | 13 | 62.7 | $-0.11t^2 + 6.38t - 0.39$ |
| 1.7 | 13 | 65.4 | $-0.22t^2 + 7.81t + 0.36$ |
| 1.7 | 11 | 53.7 | $-0.10t^2 + 5.88t + 1.19$ |
| 1.7 | 19 | 92.2 | $-0.06t^2 + 5.82t + 2.20$ |
| 1.8 | 23 | 91.2 | $-0.08t^2 + 5.49t + 4.32$ |
| 1.8 | 13 | 56.0 | $-0.11t^2 + 5.68t + 0.56$ |
| 1.8 | 12 | 56.3 | $-0.15t^2 + 6.41t + 0.84$ |
| 1.8 | 16 | 90.1 | $-0.09t^2 + 7.15t + 0.70$ |
| 1.8 | 12 | 57.1 | $-0.09t^2 + 5.66t + 1.25$ |

Table 3: Larger surf fronts.

| Wave height (m) | Total time (s) | Total distance (m) | Quadratic fit |
|-----------------|----------------|--------------------|---------------------------|
| 2.0 | 17 | 89.5 | $-0.08t^2 + 5.89t + 2.35$ |
| 2.0 | 12 | 55.9 | $-0.13t^2 + 6.15t + 1.06$ |
| 2.0 | 21 | 90.1 | $-0.11t^2 + 6.58t - 0.08$ |
| 2.0 | 28 | 136.5 | $-0.09t^2 + 7.55t - 0.68$ |
| 2.2 | 28 | 135.6 | $-0.08t^2 + 7.16t - 0.64$ |
| 2.5 | 20 | 93.5 | $-0.09t^2 + 6.47t + 1.16$ |
| 2.5 | 22 | 101.4 | $-0.05t^2 + 5.63t + 0.08$ |
| 2.5 | 14 | 70.9 | $-0.11t^2 + 6.57t + 1.19$ |

Since the rides were performed at various beaches, at various tides, on waves of different heights, speeds and types, the excellent quadratic fit for d as a function of t means that the second derivative is constant, which clearly suggests that all the broken surf fronts investigated within the surf zones were retarded uniformly.

4 Discussion

There appear to have been no previously recorded experiments on the retardation behaviour of surf fronts due to a large range of waves on real beaches. This retardation is known to be caused by the reduction in water depth as any broken or unbroken wave approaches the shore. But the details of this retardation for broken surf fronts has never before been known. The verification of a quadratic expression for d as a function of t for each ride produces two useful coefficients. The negative acceleration during each ride is predicted by twice the coefficient of t^2 . It is frequently a different value for two different rides on the same beach at the same tide. Even when the bathymetry is virtually the same, this is to be expected because the particular wave ridden is a combination of many wave trains arriving at the surf zone at

that moment. Further evidence of the variability of the retardation from one wave to the next is indicated when one surf front overtakes another (doublers). It will also depend on the water movement in the trough ahead of each surf front, which is influenced by the immediately preceding surf fronts in the set. Furthermore, this uniform retardation behaviour is still present for both spilling and plunging wave breaks.

In addition, the speed of a wave at its breaking point is given by the coefficient of t . Again this differs from one ride to the next on the same beach. Surfers know that the larger waves in a variable set of successive waves break further from the shore than smaller ones, and hence are travelling at a greater speed when they break. For some faster and larger waves recorded at break point in the Tables 1, 2 and 3, the distance travelled by the surf front, before it decays, seems relatively small. This occurred when there was a channel between the outer surf zone and the inner (shorebreak) surf zone preventing the front being ridden all the way to the shore. The average breaking speeds for the small, medium and larger waves respectively are 3.9, 5.7 and 6.5 m/s. These will be used in developing the application below.

5 Application

The new SHR system considers seven basic surf characteristics which have to be rated individually by an observer at the water's edge. One of these is the surf zone width, which has to be determined in 20 m intervals since each 20 m or part thereof of surf zone contributes a rating of one. Estimating the width of an extensive surf zone or an outer surf zone beyond a channel is not directly easy from the water's edge. However given that surf fronts are uniformly retarded provides us with a useful heuristic. An alternative formula for uniformly accelerated motion equates the distance travelled to the product of the time taken and the arithmetic mean of the initial and final velocities. Now the final velocity is approximately 2 m/s for all surf fronts just before they decay, and the average initial velocities have been noted for the three

groups of surf fronts. Hence the surf zone width is estimated approximately as $3T$ for small surf and $4T$ for medium surf, where T is the total time for a ride. These produce the correct rating for 21 of the 23 small surf rides and 34 of the 43 medium surf rides. The remainder differ by one rating value only. For waves 2.0 m and higher in table 3 the expression $4.5T$ produces six correct ratings from the eight rides, suggesting that for simplicity we use $4T$ for waves up to 2.5 m and $5T$ for higher waves. The heuristic multiplication factor will be even greater for extremely high waves. Such waves break at speeds of 10 m/s or more, as evidenced by the fact that a jet ski has to be used to tow surfboard riders onto these waves. We have not been able to collect data in these extreme cases. For our purposes, as most surf lifesaving competitions are held in waves up to 3.0 m, the zone width rating can now be simply determined using a stopwatch.

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