Investigating the effect of changing the decay coefficient in an activated sludge model

S. S. Alsaeed[1] M. I. Nelson[2] M. Edwards[3]
A. H. Msmali[4]

(Received 18 January 2022; revised 14 September 2022)

Abstract

The activated sludge process (ASP) is widely used to treat both domestic and industrial wastewaters. The main disadvantage of the ASP is the expense of disposing of excess sludge, which can account for between fifty to sixty percent of the operating costs of a treatment plant. We examine a model for the ASP in which the biochemical processes are represented by a simplified version of the well known activated sludge model No. 1. A promising method to decrease sludge production is to increase sludge biodegradability. A variety of experimental methods exist to do this, such as enzyme treatment, ozonation, heat treatment, and ultrasound. We investigate the effect of increasing the decay coefficient upon two important process variables: the chemical oxygen demand and the total suspended solids.

DOI:10.21914/anziamj.v63.17091, © Austral. Mathematical Soc. 2023. Published 2023-03-29, as part of the Proceedings of the 15th Biennial Engineering Mathematics and Applications Conference. ISSN 1445-8810. (Print two pages per sheet of paper.) Copies of this article must not be made otherwise available on the internet; instead link directly to the DOI for this article.

Contents

1 Introduction

The activated sludge process (ASP) is the most widely used biological wastewater treatment method for domestic and industrial wastewaters [9]. Figure 1 shows the most basic reactor configuration: an aerated bioreactor and a settling unit. The latter concentrates the solid components before recycling them back into the biological reactor. This recycling is critical as microorganisms are contained inside the solid and it ensures that the biological reactor maintains a larger concentration of microorganisms than would otherwise be the case. The creation of enormous amounts of sludge is one of the main drawbacks of the ASP [1].

The activated sludge model No. 1 (ASM 1) [5] is a widely used model of the ASP. It describes nitrogen and chemical oxygen demand within suspended-growth waste-treatment processes. In order to use the model the influent must be characterised by determining the values of thirteen state variables. Furthermore, the model contains twenty parameters and their values must be estimated by calibration.

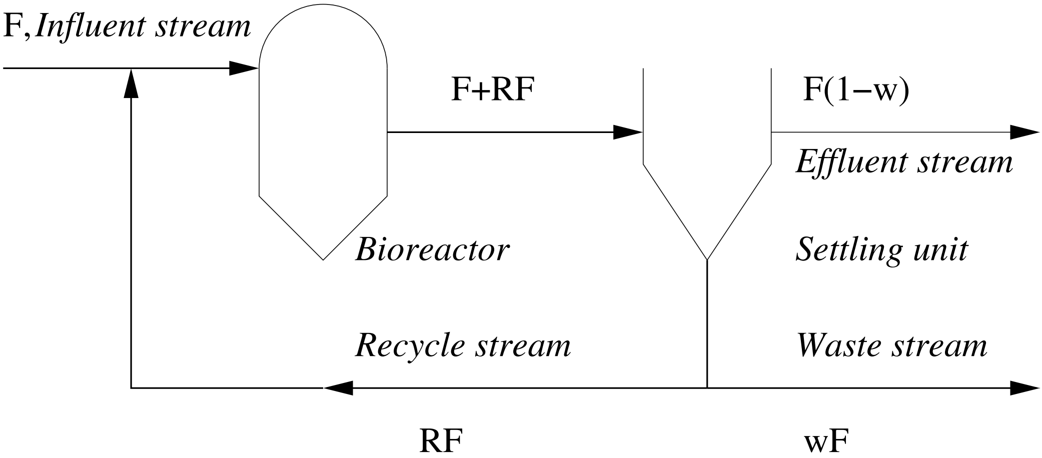


Figure 1: A single bioreactor connected to a settling unit [7] where F is the flow-rate through the bioreactor, R is the recycle ratio, and w is the fraction of the recycle stream that is wasted.

We study a simplification of the ASM 1 which excludes processes involving nitrogen. This simplification was suggested shortly after the introduction of the ASM 1 [2], however the simplified model has not been investigated. We investigate how adjusting the biomass decay changes the soluble substrate concentration and the total suspended solids inside the bioreactor. Hao et al. [4] provide the motivation for our approach in their investigation of viral infections to reduce sludge production. When their approach is applied to our model, the viral infections increase the decay rate. Future advances in biotechnology may allow the decay parameter to become a tuneable parameter. We therefore investigate the effect of both increasing and decreasing this parameter. We find that the behaviour of the reactor in response to these changes depends critically upon the value of the soluble substrate concentration in the influent stream.

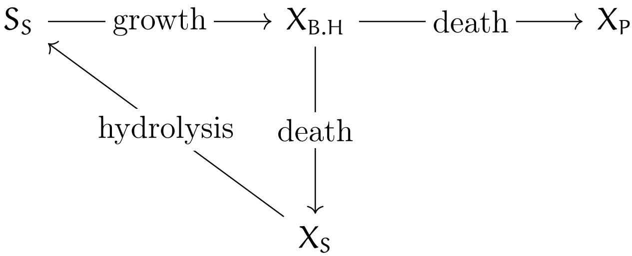


Figure 2: Overview of the biochemical processes.

2 Assumptions of the model

The simplified model used here includes three biological processes (Figure 2). In the first process the slowly biodegradable particulate substrate X_s is slowly hydrolysed by the heterotrophic biomass $X_{B,H}$ to produce the soluble substrate S_s.

In the second process the heterotrophic biomass $X_{B,H}$ grows by consumption of the soluble substrate S_s. This process is sensitive to the concentration of soluble oxygen S_O, with oxygen acting as a 'switch', turning the process on and off. When the switch is 'on' the dissolved oxygen concentration has minimal effect on the process rate.

The final process is biomass decay. This converts the heterotrophic biomass $X_{B,H}$ into a combination of nonbiodegradable particulates X_P and biodegradable particulate substrate X_s. The hydrolysis of the latter produces the soluble substrate S_s (first process), which is the growth substance for the heterotrophic biomass (second process).

3 Model equations

In the model equations (2)–(6) for a reactor of volume V with flow-rate F of the influent stream, subscript 'in' refers to the concentration of some material in the influent stream and subscript 'max' indicates the maximum possible concentration. In addition, all model equations are dependent on the reaction rates

$$M_2 = \frac{S_s}{K_s + S_s}\,, \qquad M_{8h} = \frac{S_O}{K_{O,H} + S_O}\,, \qquad k_{sat} = \frac{X_s}{K_X X_{B,H} + X_s}\,, \tag{1}$$

where K_s is the substrate half-saturation coefficient of the heterotrophic biomass $X_{B,H}$, $K_{O,H}$ is the oxygen half-saturation coefficient of the heterotrophic biomass, and K_X is the half-saturation coefficient for hydrolysis of the slowly biodegradable particulate substrate X_s. The three rates are Monod kinetic terms, where M_2 is for the biodegradable soluble substrate S_s, M_8 is for the soluble oxygen S_O with respect to the heterotrophic biomass, and k_{sat} is for the saturation kinetics.

For time t, the rate of change in the concentration of the soluble substrate is

$$V\frac{dS_s}{dt} = F(S_{s,in} - S_s) - \frac{\mu_{max,H}}{Y_H} M_2 M_{8h} X_{B,H} V + k_h k_{sat} M_{8h} X_{B,H} V\,, \tag{2}$$

where Y_H is the heterotrophic yield coefficient, k_h is a hydrolysis coefficient, and $\mu_{max,H}$ is the maximum specific growth rate for heterotrophs. The terms on the right-hand side are, respectively, the change in mass due to the flow of the influent stream through the reactor, the decrease in mass due to growth of the biomass, and the increase in mass due to hydrolysis of slowly biodegradable particles.

The rate of change in the concentration of the heterotrophic biomass is

$$V\frac{dX_{B,H}}{dt} = -FX_{B,H} + R(C-1)FX_{B,H} - b_H X_{B,H} V + \mu_{max,H} M_2 M_{8h} X_{B,H} V\,, \tag{3}$$

where C is the concentration factor for the settling unit, R is the recycle ratio of the settling unit, and b_H is the heterotrophic decay coefficient. The terms on the right-hand side are, respectively, the change in mass due to the flow of the influent stream through the reactor, the change in mass due to the flow of the recycle stream through the reactor, the decrease in mass due to death of the biomass, and the increase in mass due to growth of the biomass.

The rate of change in the concentration of the biodegradable particulate substrate is

$$V\frac{dX_s}{dt} = F(X_{s,in} - X_s) + R(C-1)FX_s + (1-f_p)b_H X_{B,H} V - k_h k_{sat} M_{8h} X_{B,H} V\,, \tag{4}$$

where f_p is the fraction of biomass yielding particulate products. The terms on the right-hand side are, respectively, the change in mass due to the flow of the influent stream through the reactor, the change in mass due to the flow of the recycle stream through the reactor, the increase in mass due to death of the biomass, and the decrease in mass due to hydrolysis.

The rate of change in the concentration of soluble oxygen is

$$V\frac{dS_O}{dt} = F(S_{O,in} - S_O) + VK_{L,A}(S_{O,max} - S_O) - \frac{(1-Y_H)}{Y_H}\mu_{max,H} M_2 M_{8h} X_{B,H} V\,, \tag{5}$$

where $K_{L,A}$ is the oxygen transfer coefficient. The terms on the right-hand side are, respectively, the change in mass due to the flow of the influent stream through the reactor, the change in mass due to the use of aerators in the reactor, and the decrease in mass due to growth of the biomass.

Finally, the rate of change in the concentration of nonbiodegradable particulate products is

$$V\frac{dX_P}{dt} = -FX_P + R(C-1)FX_P + f_p b_H X_{B,H} V\,. \tag{6}$$

The terms on the right-hand side are, respectively, the change in mass due to the flow of the influent stream through the reactor, the change in mass due to the flow of the recycle stream through the reactor, and the increase in mass due to death of the biomass.

Two common ways to characterise the performance of a treatment plant are the chemical oxygen demand (COD), which characterises the organic content of a wastewater, and the total suspended solids (TSS), which characterises the amount of sludge. The chemical oxygen demand in the influent stream, the reactor, and the effluent stream are, respectively,

$$\mathrm{COD}_{\mathrm{in}} = S_{s,\mathrm{in}} + X_{s,\mathrm{in}}\,, \tag{7}$$

$$\mathrm{COD} = S_s + X_{B,H} + X_s + X_P\,, \tag{8}$$

$$\mathrm{COD}_e = S_s\,. \tag{9}$$

The total suspended solids (with units of SS/L) in the influent stream, the reactor, and the wastage stream are, respectively,

$$\mathrm{TSS}_{\mathrm{in}} = c_1 X_{s,\mathrm{in}}\,, \tag{10}$$

$$\mathrm{TSS} = c_1\left(X_s + X_P\right) + c_2 X_{B,H}\,, \tag{11}$$

$$\mathrm{TSS}_w = C\,\mathrm{TSS}\,, \tag{12}$$

where the parameters c_i are conversion factors. The total suspended solids in the effluent stream is zero because the settling unit is assumed to capture all particulates. The parameter values used in our study are typical values for a domestic wastewater system [8].

European Union directive (91/271 'Urban wastewater') [3] fixes the maximum chemical oxygen demand allowed in the effluent of small sized wastewater treatment plants to

$$\mathrm{COD}^{\mathrm{max}} = 125\,\mathrm{mg/L}\,. \tag{13}$$

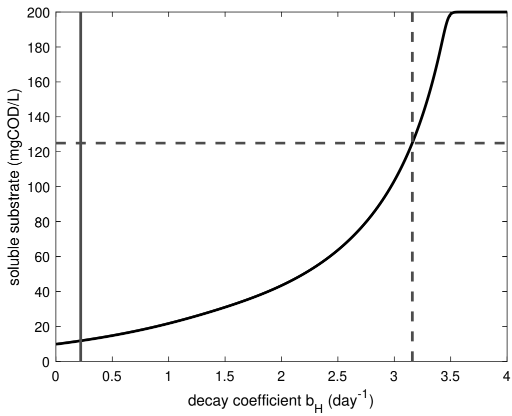
A plant must operate with the COD in the effluent stream below this value. The default value of the decay parameter is $b_H = 0.22$ day^{-1}; this is a typical value for our application [6, 10]. A key experimental parameter is the residence time, which is defined as the volume of the reactor divided by the feed flow rate $\tau = V/F$.

In what follows, a 'washout solution' refers to a solution for which the steady-state concentration of the heterotrophic biomass is zero. Literally, the biomass has been washed out of the reactor. The 'no-washout solution' refers to a solution for which the steady-state concentration of the heterotrophic biomass is positive, as are the values of all the other state variables. Note that the washout solution corresponds to process failure; the wastewater leaves the treatment plant as contaminated as it was when it entered. Consequently, wastewater treatment plants are not operated under conditions under which the washout solution is stable.

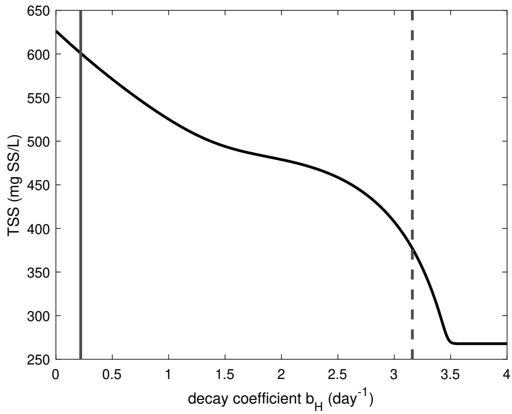
4 The effect of changing the decay parameter

Figure 3(a) shows the soluble substrate concentration S_s inside the reactor as a function of the decay coefficient b_H. Recall that the soluble substrate concentration equals the COD in the effluent stream (equation (9)), and the COD has a maximum allowable value (equation (13)), and thus the soluble substrate concentration has the same maximum value of 125 mg/L. The critical value of the decay coefficient $b_{H,cr} = 3.16$ day^{-1} is when the soluble substrate concentration attains this maximum value. When the decay coefficient is less than (larger than) the critical value, the soluble substrate concentration is below (above) the maximum value. Thus $b_{H,cr}$ is the maximum acceptable value of the decay coefficient. When the decay coefficient is large, $b_H \geqslant 3.6462$ day^{-1}, the process fails. Process failure happens when the heterotrophic biomass $X_{B,H} = 0$ and the washout steady-state solution is stable.

Figure 3(b) shows the total suspended solids inside the reactor as a function of the decay coefficient. As the decay coefficient increases the total suspended solids decreases, and thus the sludge is minimised at the critical value of b_H. When the decay coefficient is critical $b_{H,cr} = 3.16$ day^{-1}, the TSS $= 377.26$ mg SS/L, but for the default value of the decay coefficient $b_H = 0.22$ day^{-1}, the TSS $= 601.5$ mg SS/L. Thus the minimum amount of sludge is a reduction of 37.3% of the sludge obtained with the default b_H.



(a) Soluble substrate concentration inside the bioreactor.



(b) Total suspended solids inside the bioreactor.

Figure 3: Variation of the soluble substrate concentration S_s and total suspended solids TSS with the decay coefficient b_H. The solid vertical line is the default value of the decay coefficient. The horizontal dashed line is the maximum allowed chemical oxygen demand in the effluent stream (equation (13)) and the vertical dashed line is the decay coefficient at that maximum. Parameter values: $\tau = 0.15$ days; $\text{COD}_{\text{in}} = 300\,$mg COD/L; $S_{\text{s,in}} = 200$ mg COD/L; $X_{\text{s,in}} = 300 - S_{\text{s,in}}$

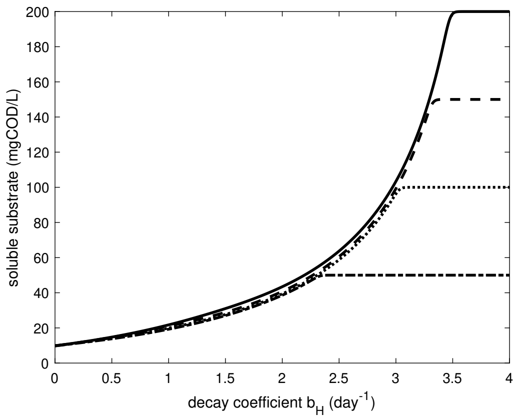
5 The effect of changing the soluble substrate concentration in the feed

We now investigate the behaviour of the soluble substrate and the total suspended solids in the reactor as a function of the decay coefficient for four values of the soluble substrate concentration in the influent stream $S_{s,in}$. We fix the chemical oxygen demand of the influent stream $\text{COD}_{in} = 300\,\text{mg/L}$. Again recall that the chemical oxygen demand in the effluent stream equals the soluble substrate concentration inside the reactor (equation (9)) and it has a maximum allowable value of $125\,\text{mg COD/L}$ (equation (13)).

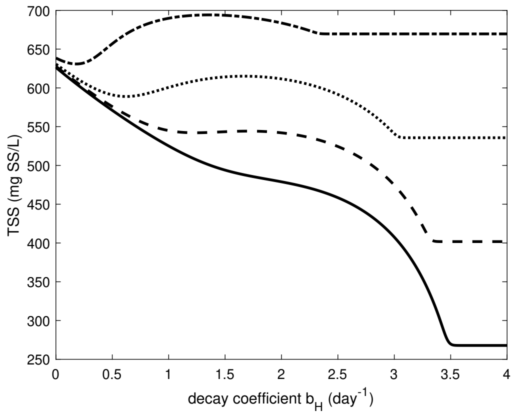
Figure 4(a) shows the soluble substrate concentration S_s as a function of the decay coefficient b_H. When the soluble substrate in the feed $S_{s,in} = 50$ or $100\,\text{mg COD/L}$ the steady-state soluble substrate concentration in the reactor is *always below* $125\,\text{mg COD/L}$. Consequently, the chemical oxygen demand in the effluent stream is always below the maximum allowed value and there is no longer a critical value of the decay coefficient. In contrast, when $S_{s,in} = 150$ or $200\,\text{mg COD/L}$ there *is* a critical value for the decay coefficient, which is $b_{H,cr} = 3.19$ or 3.16 day^{-1}, respectively.

Figure 4(b) shows the total suspended solids TSS as a function of the decay coefficient. When the soluble substrate in the feed is either $S_{s,in} = 150$ or $200\,\text{mg COD/L}$ the TSS is a decreasing function of the decay rate and consequently, the minimum TSS occurs when the decay coefficient equals its critical value. The minimum $\text{TSS} = 438.12$ or $377.26\,\text{mg SS/L}$ for critical decay coefficient $b_{H,cr} = 3.16$ or $3.19\,\text{day}^{-1}$, respectively. For the default value of the decay coefficient $b_H = 0.22\,\text{day}^{-1}$ the $\text{TSS} = 603.32\,\text{mg SS/L}$ or $600.86\,\text{mg SS/L}$ when the feed concentrations are $S_{s,in} = 150$ and $200\,\text{mg SS/L}$, respectively. The minimum values of the TSS reduce the default TSS by 37.3% or 27.3%, respectively.

When the soluble substrate in the feed is $S_{s,in} = 50$ or $100\,\text{mg COD/L}$ and there is no critical decay coefficient to define from the soluble substrate concentration S_s, we instead consider a critical concentration dependent on



(a) Soluble substrate concentration inside the bioreactor



(b) Total suspended solids inside the bioreactor.

Figure 4: Variation of the soluble substrate concentration S_s and total suspended solids TSS with the decay coefficient b_H and the influent soluble substrate concentration $S_{s,in} = 200$ mg COD/L (solid-line); 150 mg COD/L (dashed line); 100 mg COD/L (dotted line); and 50 mg COD/L (dash-dotted line). Parameter values: $X_{s,in} = 300 - S_{s,in}$, $\tau = 0.15$ days.

the total suspended solids. The TSS is respectively minimised on the no-washout and washout branches. However, in both cases we define the critical value of the decay coefficient as the value at the local minimum along the *no-washout branch* because the treatment plant will not be operated under conditions in which the washout solution branch is stable.

For $S_{s,in} = 50$ or $100\,\mathrm{mg\,COD/L}$ the no-washout branch minimum of the $\mathrm{TSS} = 630.86$ or $588.91\,\mathrm{mg\,SS/L}$ when the critical decay coefficient is $b_{H,cr} = 0.1813$ or $0.6219\ \mathrm{day}^{-1}$, respectively. For the default value of the decay coefficient $b_H = 0.22\,\mathrm{day}^{-1}$, for $S_{s,in} = 50$ or $100\,\mathrm{mg\,COD/L}$, respectively, the $\mathrm{TSS} = 631.3178\,\mathrm{mg\,SS/L}$ or $608.98\,\mathrm{mg\,SS/L}$, and these are reduced by 4.35% or 3.3% when the minimum TSS is attained. Evidently, when the concentration of soluble substrate in the feed is sufficiently small, increasing the decay coefficient is not an effective way to reduce the total suspended solids inside the reactor.

6 Conclusion

One of the main disadvantages of the ASP is the cost of disposing of excess sludge. A suite of promising techniques to alleviate this problem rely on increasing the biodegradability of the sludge within the reactor. To model these we take the decay coefficient b_H as a tuneable parameter and investigated how the concentrations of soluble substrate and total suspended solids inside the biological reactor depend upon its value.

We found that for sufficiently large soluble substrate concentrations in the feed there is a critical value for the decay coefficient. If the decay coefficient is below its critical value, then the chemical oxygen demand in the effluent stream is below its maximum allowed value (13). This imposes a restriction on how much the total suspended solids can be reduced.

If the soluble substrate concentration in the feed is sufficiently small, then the decay coefficient is always less than its critical value, and the chemical oxygen demand in the effluent stream is always lower than its maximum value (13).

In this case, the possible reduction in total suspended solids relative to the default value is very small.

We conclude that increasing the decay coefficient may be an effective method for reducing sludge formation, provided that the wastewater contains a sufficiently high soluble substrate concentration.

Acknowledgements SA gratefully acknowledges the award of a PhD scholarship by Jouf University (Saudi Arabia) to study at UoW. The authors thank Dr Judith Bunder for her detailed commentary on our manuscript which has lead to significant textual improvements.z

References

[1] R. T. Alqahtani, M. I. Nelson, and A. L. Worthy. "Sludge disintegration". In: *Appl. Math. Model.* 40.17–18 (2016), pp. 7830–7843. DOI: 10.1016/j.apm.2016.03.040 (cit. on p. C223).

[2] A. E. Billing and P. L. Dold. "Modelling techniques for biological reaction systems; 2. Modelling of the steady state case". In: *Water SA* 14.4 (1988), pp. 193–206. URL: http://wrcwebsite.azurewebsites.net/wp-content/uploads/mdocs/WaterSA_1988_04_504.PDF (cit. on p. C224).

[3] European Council. "Council Directive 91/271/EEC of 21 May 1991 concerning urban waste-water treatment". In: *Official J. E.U.* L135 (1991), pp. 40–52. URL: https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=celex%3A31991L0271 (cit. on p. C228).

[4] X. Hao, Q. Wang, Y. Cao, and M. C. M. van Loosdrecht. "Evaluating sludge minimization caused by predation and viral infection based on the extended activated sludge model No. 2d". In: *Water Res.* 45.16 (2011), pp. 5130–5140. DOI: 10.1016/j.watres.2011.07.013 (cit. on p. C224).

[5] M. Henze, C. P. L. Grady Jr, W. Gujer, G. V. R. Marais, and T. Matsuo. "A general model for single-sludge wastewater treatment systems". In: *Water Res.* 21.5 (1987), pp. 505–515. DOI: 10.1016/0043-1354(87)90058-3 (cit. on p. C223).

[6] S. G. Lu, T. Imai, M. Ukita, M. Sekine, T. Higuchi, and M. Fukagawa. "A model for membrane bioreactor process based on the concept of formation and degradation of soluble microbial products". In: *Water Res.* 35.8 (2001), pp. 2038–2048. DOI: 10.1016/S0043-1354(00)00461-9 (cit. on p. C228).

[7] M. I. Nelson, E. Balakrishnan, H. S. Sidhu, and X. D. Chen. "A fundamental analysis of continuous flow bioreactor models and membrane reactor models to process industrial wastewaters". In: *Chem. Eng. J.* 140.1-3 (2008), pp. 521–528. DOI: 10.1016/j.cej.2007.11.035 (cit. on p. C224).

[8] M. I. Nelson, H. S. Sidhu, S. Watt, and F. I. Hai. "Performance analysis of the activated sludge model (number 1)". In: *Food Bioprod. Process.* 116 (2019), pp. 41–53. DOI: 10.1016/j.fbp.2019.03.014 (cit. on p. C228).

[9] Y. Wei, R. T. Van Houten, A. R. Borger, D. H. Eikelboom, and Y. Fan. "Minimization of excess sludge production for biological wastewater treatment". In: *Water Res.* 37.18 (2003), pp. 4453–4467. DOI: 10.1016/S0043-1354(03)00441-X (cit. on p. C223).

[10] S.-H. Yoon and S. Lee. "Critical operational parameters for zero sludge production in biological wastewater treatment processes combined with sludge disintegration". In: *Water Res.* 39 (2005), pp. 3738–3754. DOI: 10.1016/j.watres.2005.06.015 (cit. on p. C228).

Author addresses

1. **S. S. Alsaeed**, School of Mathematics and Applied Statistics, University of Wollongong, NSW 2500, AUSTRALIA.

mailto:ssalsaeed@ju.edu.sa

2. **M. I. Nelson**, School of Mathematics and Applied Statistics, University of Wollongong, NSW 2500, AUSTRALIA.
mailto:mnelson@uow.edu.au

3. **M. Edwards**, School of Mathematics and Applied Statistics, University of Wollongong, NSW 2500, AUSTRALIA.

4. **A. H. Msmali**, College of Science, Jazan University, Jazan, SAUDI ARABIA.