An elementary expression in five variables relevant to many road accidents

T. P. Hutchinson[1]

(Received 28 January 2022; revised 18 May 2022)

Abstract

This article suggests a model for many road accidents, namely, those in which a vehicle being driven forwards encounters an obstacle such as a pedestrian, a vehicle, or a tree. An equation for speed of impact is obtained from the model. (Speed of impact is very important in determining severity of injury.) The equation is an expression in five variables. As it is elementary, it will be of interest to educators in mathematics and other subjects, as well as to road safety specialists. Two of the five variables (speed of vehicle, and distance of the obstacle) are conditions existing at the initiation of the emergency; and three are approximately termed: range of the sensing system; reaction time; and deceleration; and are properties of the system of driver and vehicle. Thus the focus is on the moment the emergency is appreciated, a few seconds before impact. The model and equation may be relevant to

DOI:10.21914/anziamj.v63.17113, © Austral. Mathematical Soc. 2022. Published 2022-06-06, as part of the Proceedings of the 15th Biennial Engineering Mathematics and Applications Conference. ISSN 1445-8810. (Print two pages per sheet of paper.) Copies of this article must not be made otherwise available on the internet; instead link directly to the DOI for this article.

some railway accidents, also. In addition, the article discusses how to push our understanding to a point some seconds earlier, in the hope of throwing light on accident causation.

Contents

1 Introduction **C31**

2 Model of what happens, including equation for speed of impact **C32**
- 2.1 Summary of the approach taken C32
- 2.2 Model of reaction to an obstacle C34
- 2.3 Equation for speed of impact C34
- 2.4 The five input quantities C35
- 2.5 The nature of the model C36
- 2.6 Railway accidents C37
- 2.7 Comments C38

3 Relevance to accident causation **C38**
- 3.1 Getting closer in time to the cause of the accident C38
- 3.2 Why is the obstacle present? C39
- 3.3 Human error C39

4 Discussion **C40**

1 Introduction

This article suggests a model—a clear description, that is—for many road accidents. Speed of impact is then derived.

Important background is that a one per cent reduction in speed at the time of human impact leads to approximately a three per cent reduction in the number of deaths [5], [2, Chap. 2]. The importance of impact speed has been known for a great many years, in a general way. Quite recently, it has been emphasised that a fractional reduction in deaths implies that many lives are saved, if the reduction applies to a lot of accidents. Furthermore, there is the multiplying factor of about three just mentioned. Thus a small reduction in impact speed—perhaps easier to achieve than accident prevention—will be worthwhile, if it applies widely.

The ideas here are principally of interest to specialists in road safety, whether in road engineering, vehicle engineering, psychology, or other contexts. However, they are simple enough to be understood by school and university students, and thus should be of interest to educators in mathematics, physics, and other relevant subjects.

Section 2 is concerned with constructing a five variable elementary model of road accidents. An equation is obtained for the vehicle impact speed, and there are discussions on the input quantities, the style of the model in which the inputs are employed, and possible relevance to railway accidents. Section 3 examines the possibility of extending our understanding to a point a few seconds earlier still, in the hope of identifying the cause of the accident.

2 Model of what happens, including equation for speed of impact

2.1 Summary of the approach taken

The approach taken is to consider the moment of impact, and to ask how the impact arose from the situation a couple of seconds before that. The model is one of reaction to the emergency of an obstacle being in the path of one's vehicle. The approach is detailed in the following.

- Select the class of accidents in which a vehicle being driven forwards strikes an obstacle (such as a pedestrian, a vehicle, or a tree).
- Decide that it is useful to know the impact speed because it is important

in determining severity of injury. The effect of speed is direct in the case of pedestrian impact. For vehicle occupants, speed has its effect via vehicle velocity change and then via the relative velocity of occupant and vehicle interior when contact occurs.

- Model the selected class of accidents to derive an equation for speed of impact; impact speed may be zero, meaning that impact is avoided.

The equation of the model is quite an elementary one, employing five input variables. It is sufficiently elementary that it could appear early in a course on road accidents. Hutchinson [2] provides a full account developed from earlier publications [3, 1].

There is not a strong tradition of modelling what happens in road accidents, perhaps because a great variety of events and sequences of events can and do happen. The model described below was originally proposed in the context of a vehicle equipped with an AEB (autonomous emergency braking) system [3], but might also describe a vehicle with a driver. The model refers to the last few seconds before impact, after the vehicle or driver notices an obstacle ahead. It does not demand much: there is no tracking of the movement of a potential obstacle, for example. There are other models of the same general type, though often tailored to a specific context relevant to autonomous emergency braking, autonomous weak braking, or autonomous steering [1, 2].

The equation that is obtained from the model could have been written down 100 or 150 years ago, and it may have been. However, the only similar equation known to the present writer was developed by Wooller [8, Fig. 5]. Wooller's equation is a simplification, in that it uses "distance at which a pedestrian in the path of a vehicle becomes visible to the driver", rather than the distance itself and the range of the sensing system independently.

Instead of a distance, a model might instead use a time. That is, something would happen (e.g., initiation of braking) contingent on the vehicle being within some small time of colliding with the obstacle. Distance seems appropriate for a situation in which collision is imminent and strong action is needed. Nevertheless, many things are an obstacle to a vehicle at one moment or another, and then cease being an obstacle. For example, they move out of the vehicle's path, or the vehicle's driver steers or brakes to avoid them. There is high danger when there is insufficient time to move away, or for the driver to react. For example, Suzuki et al. [7, Table 1] have braking initiated when the time to collision is sufficiently small, and braking may be one of two strengths.

2.2 Model of reaction to an obstacle

The following may be the simplest possible theory of road collisions. The driver (or an autonomous vehicle) reacts to an obstacle that is directly ahead.

> A vehicle is being driven normally at speed v. If there is an obstacle at a distance x directly ahead and within a distance d, then emergency braking with deceleration a will begin after time t.

Models are simplifications, and are sometimes wrong. Thus we should not ask if the model is correct. Instead, it is intended to be sufficiently credible to be useful for calculating results, and sufficiently simple to be understood, though perhaps accompanied by lack of realism. Factors that are absent include what the obstacle is, whether there is any obstacle nearby but not directly ahead, and whether an actual or potential obstacle is moving.

2.3 Equation for speed of impact

Impact speed is a very important factor in determining injury. The stated model leads to an explicit expression for impact speed, which is convenient. One of the familiar equations of motion under constant acceleration is used. The result involves the five quantities mentioned above (v, x, d, t, a). The square of the impact speed is

$$u^2 = \max\{0, v^2 - 2a \max[0, \min(x, d) - vt]\}. \tag{1}$$

The relevant equation of motion is adapted to introduce t and either x or d, and to reflect the ranges of validity of expressions. The starting point is $u^2 = v^2 - 2as$. This statement connects constant acceleration a, the distance over which it acts s, and the squares of the initial and impact speeds, v and u, respectively. To derive equation (1) the following points are applied.

- If driving towards a stationary obstacle, then distance d is needed; if an obstacle comes into the vehicle's path from the side, then distance x is needed. The distance $\min(x, d)$ refers to the distance of the obstacle when the need for reaction is appreciated. The subtraction of vt reflects the distance travelled during effluxion of reaction time.
- The distance $\max[0, \min(x, d) - vt]$ allows for the possibility that the distance to the obstacle may not be sufficient for reaction time to expire and reaction to start.
- The outer max function allows for the possibility that the distance to the object is sufficient for the vehicle to stop.

The derivation of equation (1) assumes that the obstacle does not itself move position. In contexts such as the reconstruction of a specific crash, relative position and movements of obstacle and vehicle are important, of course.

This model and equation give a definite answer to a definite question. The advantages of simplicicity and broad relevance are not easily found elsewhere.

2.4 The five input quantities

Equation (1) tells us what will affect impact speed: v, x, d, t, a. Even if the root cause of the event has not been eliminated, changing these quantities may mitigate or prevent the impact. There will not be an effect if there is insufficient time to react, or when distance is sufficient to stop. There is likely to be some knowledge about which of the five variables are likely to be affected by familiar road engineering or vehicle engineering measures.

Time t might be referred to as reaction time. Note, though, that it more accurately refers to the total time from when there is an obstacle directly ahead and within a distance d to when deceleration a begins. In some circumstances one may wish to distinguish between components such as detection, decision, braking system lag, and so on. The initial deceleration prior to observation of the obstacle is zero, followed by an instantaneous jump to strength a, so there is no gradual increase.

Suppose a vehicle drives into a stationary obstacle. This may be a case of failure to see at a sufficient distance to stop, or of failure to react sufficiently early to stop. In the case of a real accident, it may be impossible to distinguish between these (that is, between d being too small and t being too large).

The quantities d, t, and a are properties of the system of vehicle and driver. It may be wished to estimate them experimentally, with v and x being under experimental control. Impact speed might be measured, and this would have the advantage of examining something of great importance for injury severity. However, if the model is correct, the equation is a consequence. And if the model is approximately correct, then the equation is the natural one to use as an approximation. Consequently, rather than examine impact speed, it may be preferable to compare direct measures of the behaviour of the driver and vehicle with what the model proposes.

2.5 The nature of the model

It is important to select a class of events. In the present case, these are accidents in which an obstacle appears in front of the vehicle. Others—for example, those in which loss of control of the vehicle is the initial reason for the accident—are outside what is being considered.

It is important to select the issue to be addressed. In the present case, that is the reaction (to an obstacle), and the consequences in terms of impact speed. Focus is on a vehicle or a driver that reacts to an emergency, rather than on the vehicle or person that creates the emergency, and the question of how the obstacle came to be in the path of the vehicle is outside what is being considered. For those issues, see Section 3.

Other types of accident are likely to need other models and other equations. These may include scenarios in which a vehicle is being driven forwards in an abnormal manner, or is being driven backwards, or is being driven with some sideways component in the velocity (as when changing lanes), or when the first event is loss of control, or the first event is overturning, or there are multiple successive emergencies and perhaps multiple collisions. Equations are available describing loss of control when cornering, and overturning. Even for the type of accident under discussion, there may be subsets for which chief concern lies more than a second or two before impact—for example, anticipation of traffic events is often very difficult, but in busy traffic moving at quite high speed, observation of brake lights several vehicles ahead helps prevent crashes.

2.6 Railway accidents

Railway accidents of the type discussed here sometimes occur: one train runs into another, or into an obstruction. There may be a formal enquiry, and a report published. Emergency braking of a train is much weaker than of a road vehicle, and the time from the beginning of the event to the impact is likely to be much longer. Thus it may be possible to establish durations of the stages of the event with quite high relative accuracy.

The effort that goes into a train accident investigation, and the quite long duration of the development of the accident, suggests that it may be desirable and practicable for a train accident report to be explicit about the quantities identified in this article—v, x, d, t, a. It may be possible to estimate these, and to say whether anything could have been done to improve them and reduce the impact speed.

2.7 Comments

The style of the proposed model is clear and definite. GIVEN conditions are wwww, IF it happens that xxxx, THEN action yyyy is taken, AND CONSEQUENTLY, impact speed is zzzz. The focus is on some selected set of accidents, and on a moment very close to impact.

The model and the equation (1) may be useful beyond the stated conditions. (a) The vehicle might not be 'being driven normally'. For example, the driver may be impaired, or the vehicle may not have right of way, or the vehicle may be out of control. Even so, an obstacle directly ahead might trigger emergency braking, with the equation for impact speed being relevant. (b) Action taken might be weak braking, rather than emergency braking.

Suggestions for other models [1], [2, Chap. 12] (and references therein) might be based on specification of (a) a small number of states of the vehicle (e.g., normal driving, braking, stationary), (b) rules for transitioning between states, and (c) a dependent variable (e.g., impact speed).

3 Relevance to accident causation

3.1 Getting closer in time to the cause of the accident

Section 2 identifies what part the reacting vehicle plays in the impact, if any. But the reacting vehicle is not usually considered to be the cause of the accident. To understand events properly, it is desirable to take another step, to a few seconds prior to the obstacle being in the path of a vehicle, and look for the cause of the accident at that time. Note that the idea of 'causes' of accidents is somewhat controversial: for perhaps the last 60 years, multiple 'contributory factors' has been the preferred idea.

For the class of accidents being discussed, the natural question to ask is, why was the obstacle present? Human error will very likely provide a large set of possible reasons, and it might be asked, why was the mistake made (assuming the obstacle was present deliberately and consequent upon some mistake)?

Some lists are given in the following subsections, including reasons for the presence of an obstacle, and contexts for human error. In effect, these subsections propose disaggregation of road accidents as a necessary preliminary to engaging with the idea of causation. Consideration of causes is unlikely to be fruitful if the population of accidents is too diverse, but may be more successful once a narrower set of accidents has been defined.

3.2 Why is the obstacle present?

Considering why the obstacle is in the path of the vehicle, there are four main possibilities. (a) Human error: the obstacle is present deliberately, but as the result of a mistake. (b) The obstacle is there deliberately and perhaps without any mistake, but in some sense has become stuck. Such an obstacle is likely to be present for at least some seconds, and may be avoidable. (c) The obstacle is there deliberately and has a right to be there, as when the vehicle loses control and strikes an object or a vehicle on or off the road, or runs into the back of the vehicle ahead. (d) The obstacle's presence is inadvertent, or occurs for a reason that cannot be considered a normal mistake (many subcategories, each fairly uncommon, could be listed).

3.3 Human error

Probably the most frequent reason for an obstacle being present is that it is there deliberately, perhaps because of some sort of error. The individual responsible for the obstacle (pedestrian, driver, or rider) may have perceived a gap in the stream of traffic, and accepted that gap, in order to join the traffic stream, cross the traffic stream, or use that lane of the road to overtake.

Accidents should be disaggregated according to criteria described below. It is necessary to be clear about the type of crash under discussion—an example is that of Section 2, an obstacle being in the path of a vehicle. For that example, distinguish between error by the person without right of way and error by the person with right of way, due to a mistake about who has right of way. Also, restrict attention to short-lasting actions; consider separately relatively long-lasting states such as speeding.

Then classify actions in three ways. (a) The person is taking the initiative, or is reacting. (b) The action is normal, or is unexpected (an act that is intended to promote safety may be dangerous if it is unexpected). (c) How much thought precedes the action. For example, approximately one second might be time for a reaction but not for a thought, two seconds might be time for a thought and a decision, ten seconds might be time for thoughts and perhaps planning.

Also, distinguish (a) errors of omission from (b) errors of commission.

A psychological classification—perhaps referring to attentional, sensory, judgment, decision-making, and execution errors—is likely to be both interesting and useful. However, the view taken here is that first the totality of road accidents needs to be split into subsets.

Certain safety-related quantities that are available to the human visual system may be relevant to the decisions of the person who accepts a gap (and perhaps creates an emergency) and the driver who reacts to the emergency [4, 6]. These might be components of some future discussion on gap acceptance and human error.

4 Discussion

Several aspects of the study of road accidents have been highlighted. (a) A sequence: select a class of accidents, decide on what it is useful to know, model the selected class, and derive an equation for the desired quantity. (b) For accidents in which a vehicle being driven forwards encounters an obstacle, an elementary equation for impact speed is given. This equation is in terms of the speed of the vehicle, distance of the obstacle, range of the sensing system, reaction time, and deceleration. (c) Concerning reasons for human error by a road user who creates an obstacle in the path of another road user, it is argued that it is necessary to disaggregate the problem a great deal before any progress with psychology can be expected.

What of the future? (a) As noted at the end of Section 2.4, the writer's opinion is that comparison of elements of the model with data (from a driving simulator, perhaps), is likely to be more fruitful than comparison of the equation with data. (b) It is not clear whether the approach to human error is useful with accident data. It might be that disaggregation is practicable, but psychological interpretation is still too distant from what is recorded.

For the specific model discussed, several advantages are clear: sufficiently credible; sufficiently simple; it implies an explicit expression for impact speed; the expression involves five quantities that common sense tells us are important; it is relevant to a great many accidents; it is a base that can generate better models; and it is a step towards understanding causes of accidents. These advantages should appeal to both road safety specialists and to school and university students interested in the subject.

Acknowledgements The Centre for Automotive Safety Research, University of Adelaide, is supported by the South Australian Department for Infrastructure and Transport. The views expressed are those of the author, and do not necessarily represent those of the University of Adelaide or the funding organisation.

References

[1] T. P. Hutchinson. "A method of constructing models of reaction to an imminent road crash". In: *Traf. Eng. Control* 57 (2016), pp. 97–103 (cit. on pp. C33, C38).

[2] T. P. Hutchinson. *Road Safety Theory*. Published online. 2018. URL: http://RoadSafetyTheory.com (cit. on pp. C31, C33, C38).

[3] T. P. Hutchinson. "The theory of reduction of impact speeds". In: *Traf. Eng. Control* 56 (2015), pp. 177–180 (cit. on p. C33).

[4] D. N. Lee. "A theory of visual control of braking based on information about time-to-collision". In: *Perception* 5 (1976), pp. 437–459. DOI: 10.1068/p050437 (cit. on p. C40).

[5] D. F. Moore. "Minimization of occupant injury by optimum front-end design". In: *Society of Automotive Engineers Technical Report No. 700416* (1970). DOI: 10.4271/700416 (cit. on p. C31).

[6] D. Stewart, C. J. Cudworth, and J. R. Lishman. "Misperception of time-to-collision by drivers in pedestrian accidents". In: *Perception* 22 (1993), pp. 1227–1244. DOI: 10.1068/p221227 (cit. on p. C40).

[7] K. Suzuki, H. Tanaka, Y. Miichi, and M. Aga. "Collision-mitigation level of collision-avoidance braking system". In: *Int. J. Vehicle Safety* 7 (2014), pp. 1–16. DOI: 10.1504/IJVS.2014.058238 (cit. on p. C34).

[8] J. Wooller. "Road traffic accidents in Adelaide and Brisbane, Australia—Excerpts from a report in preparation". In: *Proceedings, 4th Conference, Australian Road Research Board* 4 (1968), pp. 976–994. URL: http://155.212.5.248/Presto/content/Detail.aspx?ctID=MjE1ZTI4YzctZjc1YS00MzQ4LTkyY2UtMDJmNTgxYjg2ZDA5&rID=OTAx&qrs=RmFsc2U=&ph=VHJ1ZQ==&bckToL=VHJ1ZQ==&rrtc=VHJ1ZQ== (cit. on p. C33).

Author address

1. **T. P. Hutchinson**, Centre for Automotive Safety Research, University of Adelaide, South Australia 5005, AUSTRALIA.
mailto:timothy.hutchinson@adelaide.edu.au
orcid:0000-0002-4429-0885