MISG, mines and variability

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Abstract

In 2016, a Mathematics-in-Industry Study Group (MISG) project considered the construction of mining sequences, that is, the process connecting ore extraction with specific orders. In particular, the meeting considered the potential for using knowledge about geological variability within the ore. This article revisits this MISG project and the approach developed for thinking about the problem as the build for an order progresses. We provide new perspectives on this approach and outline possible ways for further development.

Contents

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1 Introduction

The 2016 Mathematics-in-Industry Study Group, MISG 2016, held at the University of South Australia, included a project considering the management of an open pit mine. This project was brought to the study group by Schneider Electric. The operational challenge involves matching ore extraction to demand in the most effective way [3, 2, 1].

To manage the extraction of ore, a mine's contents are divided into blocks using a spatial grid. The mean ore content of each block is predicted using a grid of drilled core samples. Potentially, the variance of the ore content can also be estimated, for instance by considering the location of the nearest core samples: the closer the location of the core sample the more accurate the estimate of the mean.

Demand for the ore comes in the form of distinct orders, called builds, that have specified tonnage and grade (percentage ore). Each of these is assembled from a number of blocks and must meet the grade to within a specified tolerance.

Once a block has been mined its grade can be found more exactly. The block is allocated towards building an order, put into a waste heap, or put into a temporary stock pile for later use. The disadvantage in using temporary stock piles is the increased transport cost. Ideally, blocks would be mined and go directly to a build or to the waste heap.

At the MISG study group, we explored various aspects of the process. We considered valid paths for ore extraction, since buried ore cannot be extricated until it is exposed by removal of the rock above. A deterministic mixed integer program was formulated. A heuristic to select blocks was evaluated—it is this aspect of the MISG investigation that is revisited here. Finally, the MISG study group simulated the build construction process itself. Further details of the MISG project and the mining process are included in the report [4].

2 Using estimated grade variance to select blocks

We outline a model developed at the MISG study group. We further illustrate and explore the approach and consider potential application of the model.

The mining objective is to select blocks in such a way so as to ensure that the build is likely to meet the required grade to within a specified tolerance. It is assumed that blocks are available for mining with a range of predicted grades and a mixture of uncertainties (variance) in those predicted mean values. Our model constructs the build sequentially, block by block. At each stage we assume that it is possible to change to use higher or lower grade blocks during the rest of the build. We compute the probability that with this modification our build will then be feasible. Different strategies for ordering the blocks in a build result in different feasibility probabilities. For preference of a strategy the feasibility probability should be as high as possible.

The approach suggested at the MISG study group was that, when constructing a build sequentially, it is generally better to incorporate the more-uncertain-grade blocks first. To illustrate this approach, consider a toy problem of a mine with blocks for a range of grades. There are two sorts of blocks: for some of the blocks we know the grade exactly but for others we do not know anything about the grade at all. To construct a two-block build of grade 50%, we preferably should only need to mine two blocks that average to this value. However, the only way to ensure that this is possible, and to use the unknown-grade blocks, is to mine an unknown-grade block first, discover the grade, and then chose an appropriate known-grade block to extract to match it. Another toy problem of a similar nature is included in the MISG report [4].

More generally, we consider a build consisting of N blocks with target grade T obtained within tolerance ϵ; that is the final grade must lie within $[T-\epsilon, T+\epsilon]$. For simplicity, we take the tolerance to be symmetrical, although, in practice it is possible that the cost of undershooting a target grade is worse than overshooting it. Further, for simplicity, the blocks have an identical mass M. (The generalisation to varying block mass is straightforward [4].) The blocks have normally distributed grades with standard deviations σ_i, $i = 1 \ldots N$. We assume that the acceptable interval for the final grade, $[T - \epsilon, T + \epsilon]$, is feasible at some probability.

Assume that n blocks are incorporated into a partial build so that their predicted mean values average the target grade. There is no adjustment to allow for the actual grades measured after excavation: such adjustments may be necessary but in our model this occurs later in the build (cf. Section 4). Two sets of measures are useful: the 'standard deviation cone' and the 'feasibility cone' (Subsections 2.1 and 2.2, respectively). These cones each consist of upper and lower values corresponding to every stage of the build. Combined, these 'cones' are used to estimate a likelihood of success at each stage of the build.

We illustrate the process of modelling a build by considering a 120-block case study. We assume that the available blocks are divided between predicted mean grades of 57%, 60% or 63% and that these grades in turn are equally divided between predictions with standard deviation 1% and 3%; that is, the block grades are of two different uncertainties in equal quantities. The target grade is 60%. The first (preferred) approach initially uses 60 blocks of the lesser-known-grade (with standard deviation 3%), followed by 60 better-known-grade ones. The second approach reverses the order so that the 60 better-known-grade blocks come first. This case study is a modification of that of the MISG study group [4]. It improves on that study as the two approaches here have equal quantities of blocks for each standard deviation.

2.1 The standard deviation cone

The standard deviation cone is constructed by taking the expected value after n blocks have been included in the build, and then either adding or subtracting the overall standard deviation. For simplicity we normalise this quantity by the total mass of the n blocks. The width of the cone is

$$s_n = \frac{\sqrt{\sum_{i=1}^{n}(M\sigma_i)^2}}{\sum_{i=1}^{n} M} = \frac{1}{n}\sqrt{\sum_{i=1}^{n}\sigma_i^2}\,. \tag{1}$$

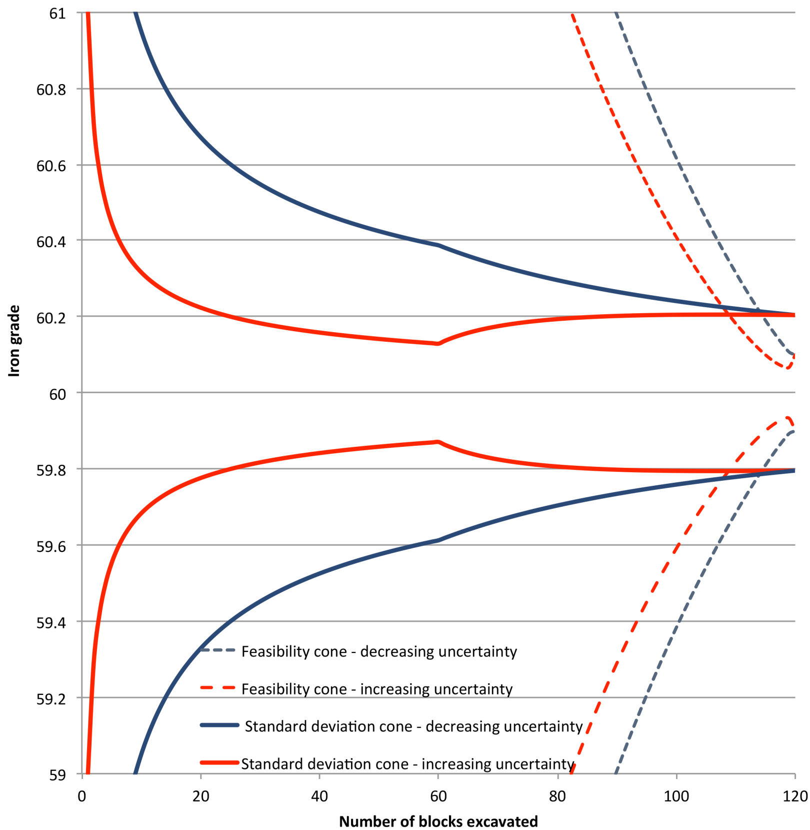
If we assemble n blocks so that their predicted mean grade is the target grade, then the standard deviation cone indicates how far we can expect the build to deviate from the target.

Figure 1 shows the standard deviation cones for the two approaches in our case study. In general, the cones converge during the build as deviations for individual blocks tend to cancel out. This is the case for the outer (blue) cone for the instance where the lesser known blocks are used first. However, for the inner (red) cone a divergence arises later in the process when the lesser-known-grade blocks are added and increase the overall deviation. The cones for the two approaches converge as they contain blocks with the same distribution of deviations from predicted grades.

2.2 The feasibility cone

The feasibility cone consists of upper bounds on the build's current grade g_n^+, above which it is impossible to achieve the target grade range at the prescribed

Figure 1: The case study's standard deviation and feasibility cones (solid and dashed lines, respectively). The blue lines are the approach using the lesser-known-grade (standard deviation 3%) blocks initially, whereas for the red lines they are used at the end.



probability (taken as 0.99 in our case study). That is, even exclusive use of the lowest predicted grade blocks G^- from $n+1$ to N overshoots the target too often. Similarly, there is a lower bound g_n^-, related to the highest grade blocks G^+. As well as the target grade bound and extreme block adjustment, to obtain the 0.99 probability the bounds include a factor 2.32 for the random normal variable 99% confidence interval [4] and

$$g_n^{\pm} = \frac{N(T \pm \epsilon) - (N-n)G^{\mp} \mp 2.32\sqrt{\sum_{i=n+1}^{N} \sigma_i^2}}{n} \,. \tag{2}$$

In the limit $n \to N$, all the blocks are incorporated into the build, and these upper and lower bounds (g_n^+, g_n^-) converge to the target bounds $T \pm \epsilon$. Some care is required. For example, unless $g_n^+ > g_n^-$, calculations involving these quantities are meaningless. We can further simplify the expressions for the bounds if the later-added blocks' standard deviations are identical to each other. Then the upper bound reduces to

$$g_n^{+} = \frac{N(T + \epsilon) - (N-n)G^{-} - 2.32\sigma\sqrt{N-n}}{n} \,. \tag{3}$$

Treating the expression (3) as a continuous function of n, the first and second derivatives can be found. In the limit as $n \to N$, both of these derivatives tend to infinity and the expression is concave upwards as it converges to $T+\epsilon$. The lower bound behaves similarly but in the opposite direction.

Figure 1 shows the feasibility cones for our case study. Both feasibility cones initially converge inwards. However, near the end of the build we see the change to divergence in the red curve when the lesser-known blocks are used to finish the build. The similar change for the blue curve is not visible as it would occur at a fraction of a block from the end ($n = 119.85$). Again, the cones for both approaches have the same final value.

2.3 The feasibility probability

As variation is governed by the normal distribution, the probability of the build, at n blocks, being within the feasibility cone is obtained as an error function. The range of likely grade values is indicated by the standard deviation cone. The argument of the error function involves the ratio of the widths of the current feasibility and standard deviation cone bounds [4]. The probability is

$$p = \operatorname{erf}\left(\frac{\epsilon N - 2.32\sqrt{\sum_{i=n+1}^{N}\sigma_i^2} + \frac{1}{2}(G^+ - G^-)(N-n)}{\sqrt{2}\sqrt{\sum_{i=1}^{n}\sigma_i^2}}\right). \qquad (4)$$

Figure 2 shows the feasibility probabilities for our case study. In both cases the probability is effectively unity until the 90th block's inclusion. After this we observe a decrease. The red line, representing the build that incorporates the more-certain-grade blocks first, decreases first and remains lower. This curve rises at the end, as a result of the increase in width of the feasibility cone discussed in Subsection 2.2. The final probability is the same for the different orderings, as a consequence of both sets of cones converging as discussed in Subsections 2.1 and 2.2.

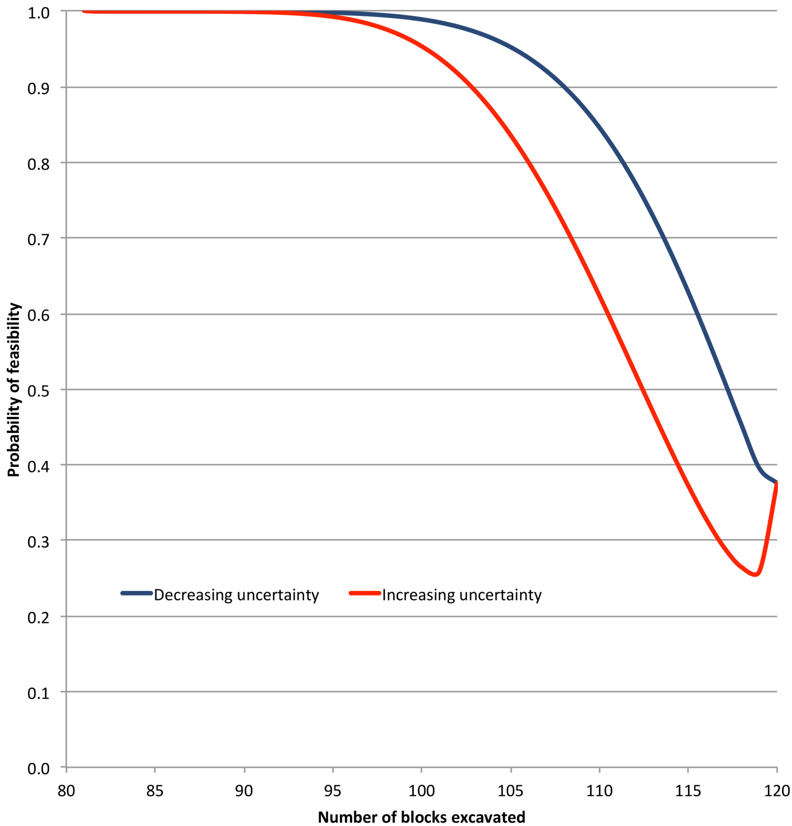
3 Ordering the blocks

As observed by the MISG group [4], it seems intuitive that using more uncertain blocks first followed by more certain ones is a better approach than using the more certain ones first and the uncertain ones at the end. Figures 1 and 2 illustrate that this is true for our case study. Although we have not been able to show that it is always optimal, we are able to argue that it is likely to be optimal in practical application.

Our measure of a strategy being better is that the feasibility probability during the build is higher (Subsection 2.3). We describe the application to our simplified model, however the arguments can be generalised.

Suppose that we are constructing a build, as in Section 2. Upon including the penultimate block, that is, at $n = N - 1$, the probability of feasibility,

Figure 2: Feasibility probability for the case study. As in Figure 1, the blue lines are the approach using the lesser-known-grade (standard deviation 3%) blocks initially, whereas for the red lines they are at the end.



equation (4), becomes

$$p = \operatorname{erf}\left(\frac{\epsilon N + \frac{1}{2}(G^+ - G^-) - 2.32\sigma_N}{\sqrt{2}\sqrt{\sum_{i=1}^N \sigma_i^2 - \sigma_N^2}}\right) = \operatorname{erf}\left(\frac{2.32}{\sqrt{2}}\,\frac{K_N - \sigma_N}{\sqrt{Q_N^2 - \sigma_N^2}}\right), \quad (5)$$

where $K_N = [\epsilon N + \frac{1}{2}(G^+ - G^-)]/2.32$ and $Q_N = \sqrt{\sum_{i=1}^N \sigma_i^2}$. Assuming that we are only varying the order of the blocks in the build, then K_N and Q_N are constants and our feasibility probability at $n = N - 1$ is a function of σ_N. Increasing σ_N corresponds to changing the order of the blocks so that the last block is one that is more uncertain. Likewise, decreasing σ_N corresponds to increasing the certainty of the last block.

For a feasible build, both the denominator and the numerator in the argument of expression (5) are positive, as are the standard deviations. Hence $K_N - \sigma_N > 0$, $Q_N^2 - \sigma_N^2 > 0$ and $\sigma_N > 0$ (assuming $N > 1$).

The derivative of the argument in expression (5) is

$$\frac{d}{d\sigma_N}\left(\frac{2.32}{\sqrt{2}}\,\frac{K_N - \sigma_N}{\sqrt{Q_N^2 - \sigma_N^2}}\right) = -\frac{2.32}{\sqrt{2}}\,\frac{Q_N^2 - \sigma_N K_N}{(Q_N^2 - \sigma_N^2)^{3/2}}\,. \quad (6)$$

This is negative when $Q_N^2 > \sigma_N K_N$ and in these circumstances increasing the uncertainty of the last block leads to a lower feasibility probability.

In general, we know that Q_N is larger than σ_N and probably much larger if N is large. However, the comparative magnitudes of K_N and Q_N are less clear. Both K_N and Q_N^2 are $O(N)$. However, in equation (5), we are interested in cases when the feasibility probability becomes low. If the second part of the argument of the function is above unity, that is, $K_N - \sigma_N \geqslant \sqrt{Q_N^2 - \sigma_N^2}$, then the probability is above 0.98. For lower probabilities $\sqrt{Q_N^2 - \sigma_N^2} > K_N - \sigma_N$ and, given that $Q_N > \sigma_N$, the condition $Q_N^2 > \sigma_N K_N$ seems reasonable.

This analysis has considered the penultimate block of a build. Earlier blocks in the build may be considered similarly by treating the partial build as if it were the complete build.

4 An approach to application

As a build is constructed, there is uncertainty in what the cumulative grade will be because the grade of each individual constituent block is unknown until it has been excavated. Furthermore, it is possible that multiple blocks will be excavated for the same build concurrently; reducing the sequential nature of the build. However, the estimated feasibility probability (4) can be used to plan when to intervene in a build to ensure it is completed satisfactorily. We illustrate such an approach by reconsidering the case study introduced in Section 2.

While constructing the 120-block build described in the case study, we now plan so to maintain the probability of feasibility above a value of 0.98. For this value the argument of the erf function in expression (4) is approximately 1.645. Using the (preferred) approach where we use the 60 more-uncertain blocks first we find that the probability falls below 0.98 immediately after $N_2 = 102$ blocks. So therefore we plan to review which blocks to use at this point (we could do this earlier).

After adding 102 blocks our new aim is to choose the remaining 18 blocks so that their expected means sum to an amount that counteracts the drift from the target that takes place in the partial build from the first 102 blocks. For example if the first 102 blocks have a mean grade of 59.7% which is below our target of 60% we choose the remaining 18 blocks to have a mean grade G_2% so that

$$102 \times 59.7 + 18 \times G_2 = 120 \times 60\,, \tag{7}$$

that is, $G_2 = 61.7$. We now regard this G_2% value as being a new target grade and apply the model in a similar way to the final 18 blocks. We have reduced the term in the denominator of expression (4) which becomes

$$p = \operatorname{erf}\left(\frac{\epsilon N - 2.32\sqrt{\sum_{i=n+1}^{N}\sigma_i^2} + \frac{1}{2}(G^+ - G^-)(N-n)}{\sqrt{2}\sqrt{\sum_{i=N_2+1}^{n}\sigma_i^2}}\right). \tag{8}$$

For the case study, this probability does not fall below 0.98 and we complete the build.

If the new feasibility probability were to again have fallen below 0.98 before the end of the build, then the process would have been repeated again.

The feasibility probability may be used in other ways. For example, if the feasibility probability assessed during a build is very high, then this could be an opportunity to use more uncertain blocks than originally planned.

5 Discussion

The calculations in Sections 2 and 3 use a number of simplifications and approximations. It is assumed that quantities involved are normal and symmetrical. As the grades of the blocks must all be positive, their distribution is not strictly normal. Further, the bounds on the target grade may be smaller below than above, as the penalty for producing a grade too low is likely to be worse than producing one that is too high. However, the approaches described can be adapted to more general cases.

In the non-symmetrical case, to generalise equation (1) the two sides of the standard deviation cone can be built separately using appropriate one-sided measures of variance and hence the width of the cone is found. The variance term in the bounds (2) are similarly adapted. Non-symmetrical target tolerance ϵ, and extreme grade blocks that are available, G^+ and G^-, are readily accommodated into equation (2). Similarly, blocks of different masses are readily accommodated, as included in the earlier MISG study [4].

We have illustrated an approach for application of the model (Section 4). This may similarly be adapted to a more general case.

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