Process capability estimation for non–normal quality characteristics: A comparison of Clements, Burr and Box–Cox Methods

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Abstract

In today's competitive business environment, it is becoming more crucial than ever to assess precisely process losses due to non-compliance to customer specifications. To assess these losses, industry is widely using process capability indices for performance evaluation of their processes. Determination of the performance capability of a stable process using the standard process capability indices requires that the underlying process data should follow a normal distribution. However, if the data is non-normal, measuring process capability using conventional methods can lead to erroneous results. Different process capability indices such as Clements percentile method and data transformation method have been proposed to deal with the non-normal

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situation. Although these methods are practiced in industry, there is insufficient literature to assess the accuracy of these methods under mild and severe departures from normality. This article reviews the performances of the Clements non–normal percentile method, the Burr based percentile method and Box–Cox method for non-normal cases. A simulation study using Weibull, Gamma and Lognormal distributions is conducted. Burr's method calculates process capability indices for each set of simulated data. These results are then compared with the capability indices obtained using Clements and Box–Cox methods. Finally, a case study based on real world data is presented.

Contents

1 Introduction

Process mean μ, process standard deviation σ and product specifications are basic information used to evaluate process capability indices. However, product specifications are different in different products [10]. A frontline manager of a process cannot evaluate process performance using μ and σ only. For this reason Juran [9] combined process parameters with product specifications and introduced the concept of Process capability indices (PCI). Since then, the most common indices being applied by manufacturing industry are process capability index C_p and process ratio for off-center process C_{pk} defined as

$$C_p = \frac{\text{Allowable process spread}}{\text{Actual process}} = \frac{U_t - L_t}{6\sigma}, \quad (1)$$

$$C_{pk} = \min\{C_{pu}, C_{pl}\}, \quad (2)$$

$$C_{pu} = \frac{U_t - L_t}{3\sigma}, \quad C_{pl} = \frac{\mu - L_t}{3\sigma}, \quad (3)$$

where U_t and L_t are the upper and lower tolerance limit respectively, and C_{pu} and C_{pl} refer to the upper and lower *one sided* capability indices. In the actual manufacturing process, μ and σ are unknown and are often estimated using historical data [14]. Note that C_p is applied to determine process capability with bilateral specifications whereas C_{pu} and C_{pl} are applied to process capability with unilateral specification.

The capability indices C_p and C_{pk} are essentially statistical measures and their interpretations rely on the validity of certain assumptions. Some of the basic assumptions of traditional process capability indices are that

- the process under examination must be under control and stable;
- the output data must be independent and normally distributed.

However, these assumptions are not usually fulfilled in practice. Many physical processes produce non-normal data and quality practitioners need to verify that the above assumptions hold before deploying any PCI techniques to determine the capability of their processes. This article discusses process capability techniques in cases where the quality characteristics data is non-normal and then compare the accuracy of these PCIs using Burr XII distribution [1] instead of the Pearson family of curves employed in the Clements percentile method [4]. For illustrative purposes, we perform a comparison study of Burr based PCI with Clements and Box–Cox [3] power transformation methods. Finally, Section 4 presents two application examples with real data.

2 Methods to estimate process capability for non-normal process

When the distribution of the underlying quality characteristics data is not normal, there have been some modifications of the conventional PCIs presented in the quality control literature to resolve the issue of non-normality. Kotz and Johnson [7] presented a detailed overview of the various approaches related to PCIs for non-normal data. One of the more straightforward approaches is to transform the non-normal output data to normal data. Johnson [5] proposed a system of distributions based on the moment method, called the Johnson transformation system. Box and Cox [3] also used transformations for non-normal data by presenting a family of power transformations which includes the square-root transformation proposed by Somerville and Montgomery [13] to transform a skewed distribution into a normal one. The main objective of all these transformations is that once the non-normal data is transformed to normal data, one then apply the same conventional process capability indices which are based upon the normal assumption. Clements [4] proposed another approach to handle non-normal data: this is called a quantile based approach and provides an easy method to assess the capability indices for non-normal data. Clements used non-normal percentiles to modify the classical capability indices.

2.1 Clements percentile method

The Clements Method is popular among quality practitioners in industry. Clements [4] proposed that 6σ in equation (1) be replaced by the lengths of interval between the upper and lower 0.135 percentage points of the distribution of X, that is, the denominator in equation (1) is replaced by $U_p - L_p$:

$$C_p = \frac{U_t - L_t}{U_p - L_p}\,, \tag{4}$$

where U_p is the upper 99.865 percentile and L_p is the lower 0.135 percentile of the observations respectively. Since the median M is the preferred central value for a skewed distribution, he also defined

$$C_{pu} = \frac{U_t - M}{U_p - M}\,, \tag{5}$$

$$C_{pl} = \frac{M - L_t}{M - L_p}\,, \tag{6}$$

$$\text{and} \quad C_{pk} = \min\{C_{pu}, C_{pl}\}\,. \tag{7}$$

The Clements Method uses the standard estimators of skewness and kurtosis that are based on third and fourth moments respectively which may not be reliable for very small sample sizes [12]. These third and fourth moments are then used to fit a suitable Pearson distribution using the data set. The upper and lower percentiles are then obtained from the selected Pearson distribution. Wu et al. [15] conducted research indicating that the Clements Method cannot accurately measure the capability indices, especially when the underlying data distribution is skewed.

2.2 Box–Cox power transformation method

Box and Cox [3] proposed a family of power transformations on a necessarily positive response variable X given by

$$X^{(\lambda)} = \begin{cases} \dfrac{X^\lambda - 1}{\lambda}\,, & \text{if } \lambda \neq 0\,, \\ \ln(X)\,, & \text{if } \lambda = 0\,. \end{cases} \qquad (8)$$

This transformation depends upon a single parameter λ that is estimated using Maximum Likelihood Estimation (MLE) [9]. The transformation of non-normal data to normal data using Box–Cox transformation is available in most statistical software packages; consequently, the users can deploy this technique directly to evaluate PCIs.

2.3 The Burr percentile method

Burr [1] proposed a distribution called Burr XII distribution to obtain the required percentiles of a variate X. The probability density function of a Burr XII variate Y is

$$f(y|c,k) = \begin{cases} \dfrac{cky^{c-1}}{(1+y^c)^{k+1}}\,, & \text{if } y \geq 0, c \geq 1, k \geq 1\,, \\ 0\,, & \text{if } y < 0\,, \end{cases} \qquad (9)$$

where c and k represent the skewness and kurtosis coefficients of the Burr distribution respectively.

Liu and Chen [12] introduced a modification based on the Clements method, whereby instead of using Pearson curve percentiles, they replaced them with percentiles from an appropriate Burr distribution. The proposed method is outlined in the following steps and also illustrated by an example with $U_t = 32$ and $L_t = 4$ in Table 1.

1. Estimate the sample mean $\bar{x}$, sample standard deviation s, skewness s_3 and kurtosis s_4 of the original sample data. Note that

$$\begin{aligned} s_3 &= \frac{n}{(n-1)(n-2)} \sum \left(\frac{x_j - \bar{x}}{s} \right)^3 \\ \text{and} \quad s_4 &= \frac{n(n+1)}{(n-1)(n-2)(n-3)} \sum \left(\frac{x_j - \bar{x}}{s} \right)^4 - \frac{3(n-1)^2}{(n-2)(n-3)} \,. \end{aligned}$$

2. Calculate standardized moments of skewness α_3 and kurtosis α_4 as

$$\alpha_3 = \frac{(n-2)}{\sqrt{n(n-1)}} s_3 \qquad (10)$$

$$\text{and} \quad \alpha_4 = \frac{(n-2)(n-3)}{(n^2-1)} s_4 + 3\frac{(n-1)}{(n+1)} \,. \qquad (11)$$

The kurtosis defined by (11) is commonly known as the *excess* kurtosis since it adjusts the ordinary kurtosis value above and below the value of +3.00 (which indicates the absence of kurtosis , that is, the distribution is mesokurtic). Negative value identifies a platykurtic distribution and positive value that of a leptokurtic distribution.

3. Use the values of α_3 and α_4 to select the appropriate Burr parameters c and k [1]. Then use the Burr distribution XII [2] to obtain the distribution of the standardized variate

$$Z = \frac{Y - \mu}{\sigma} \,,$$

where Y is the selected Burr variate, μ and σ its corresponding mean and standard deviation respectively. The means and standard deviations, as well as skewness and kurtosis coefficients, for a large collection of Burr distributions are found in tables of Burr [2], Liu and Chen [12]. From these tables, the standardized 0.00135, 0.5, 0.99865 percentiles, that is, $Z_{0.00135}$, $Z_{0.5}$ and $Z_{0.99865}$, are obtained. Obtain corresponding percentiles of X by matching the two standardized values, that is

$$\frac{X - \bar{x}}{s} = \frac{Y - \mu}{\sigma} . \tag{12}$$

4. From (12), the estimated percentiles for lower, median, and upper percentiles are

$$L_p = \bar{x} + s\, Z_{0.00135} , \tag{13}$$
$$M = \bar{x} + s\, Z_{0.5} , \tag{14}$$
$$\text{and} \quad U_p = \bar{x} + s Z_{0.99865} . \tag{15}$$

5. Calculate process capability indices using equations (4) to (7).

Instead of using moments of skewness and kurtosis as we have done here, other methods such as Maximum Likelihood, Method of Probability-Weighted Moments and Method of L-Moments [11] are also used to estimate parameters of a Burr distribution. However, our choice is determined by the fact that quality control practitioners with little background in theoretical statistics will find the estimation procedure adopted here, which is simply a moment matching process, much easier to comprehend and apply.

3 A simulation study

3.1 Comparison criteria

Different comparison yardsticks lead to different conclusions. A widely recognized yardstick for tackling the non-normality problem for PCI estimation is given by Rivera et al. [8]. They used upper tolerance limits of the underlying distributions to calculate the actual number of non-conformance items

Table 1: PCI calculations using Burr Percentile Method.

Procedure	Parameters	Calculated values
Enter specifications:		
Upper tolerance limit	U_t	32
Lower tolerance limit	L_t	4
Estimate sample statistics:		
Sample size	n	100
Mean	$\bar{x}$	10.5
Standard deviation	s	3.142
Skewness	s_3	1.14
Kurtosis	s_4	2.58
Use s_3 and s_4 to calculate:		
Standardized moment of skewness	α_3	1.12
Standardized moment of kurtosis	α_4	4.97
Based on α_3 and α_4, select c and k:		
	c	2.347
	k	4.429
Use the estimated Burr XII distribution to obtain:		
standardized lower percentile	$Z_{0.00135}$	−1.808
standardized median	$Z_{0.5}$	−0.140
standardized upper percentile	$Z_{0.99865}$	4.528
Calculate estimated 0.135 percentile using (14):	L_p	4.819
Calculate estimated median using (15):	M	10.06
Calculate estimated 99.865 percentile using (15):	U_p	24.727
Calculate CPIs using (4)–(7) :	C_p	1.40
	C_{pu}	1.49
	C_{pl}	1.15
	C_{pk}	1.15.

and equivalent C_{pk} values. Estimated C_{pk} values calculated from the data are then compared with the target C_{pk} values. A similar motivated scheme has been used as a comparison yardstick for one-sided C_{pu} by Tang et al. [9] and Liu and Chen [12] in their non-normal PCIs studies. For a target C_{pu} value, the fraction of non-confirming items from a normal distribution can be determined using

$$\text{Fraction of non-conforming parts} = \Phi(-3C_{pu}) \qquad (16)$$

where $\Phi(x)$ refers to the cumulative distribution function of the standard normal random variable [6].

In this article, the process capability index C_{pu} with unilateral tolerance limit is used as comparison criterion. Weibull, Gamma and Lognormal distributions are used to investigate the effect of non-normal data on the process capability index. These distributions are known to have parameter values that represent mild to severe departures from normality. These parameters are selected to compare our simulation results with existing results using the same parameters as Tang & Than [9], and Liu & Chen [12].

The probability density functions of Weibull, Gamma and Lognormal distributions are

$$\begin{aligned}
&\text{Weibull}(\alpha,\beta),\ \alpha>0,\beta>0:\\
&\qquad f(x|\alpha,\beta) = \frac{\alpha}{\beta}x^{\alpha-1}e^{-x^\alpha/\beta}, \quad x\geq 0.\\
&\text{Gamma}(\alpha,\beta),\ \alpha>0,\beta>0:\\
&\qquad f(x|\alpha,\beta) = \frac{1}{\gamma(\alpha)\beta^\alpha}x^{\alpha-1}e^{-x/\beta}, \quad x\geq 0.\\
&\text{Lognormal}(\mu,\sigma^2),\ -\infty<\mu<\infty:\\
&\qquad f(x|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}x}e^{-(\ln x-\mu)^2/2\sigma^2} \quad x\geq 0.
\end{aligned}$$

In our simulation study, for comparison purposes, the target C_{pu} values of 0.5, 1.0, 1.5 and 2.0 and the Weibull distribution with $\alpha = 1.2$ and $\beta = 1.0$, Gamma distribution with $\alpha = 1.0$ and $\beta = 1.0$ and Lognormal distribution with $\mu = 0$ and $\sigma^2 = 1.0$ are used. The corresponding U_t value for each distribution is

$$U_t = C_{pu}(X_{0.99865} - X_{0.5}) + X_{0.5}\,, \tag{17}$$

where $X_{0.99865}$ and $X_{0.5}$ are the designated percentiles of the corresponding distribution. For example, if $C_{pu} = 1.5$ and the underlying distribution is, say Weibull with parameter values $\alpha = 1.2$ and $\beta = 1$, then using any statistical package one obtain the two percentiles which are $X_{0.99865} = 4.8236$ and $X_{0.5} = 0.7368$ respectively. Then we use equation (17) to find the corresponding U_t which equals 6.867.

We next simulate 30 samples, each of size 100, from each distribution and follow the steps outlined below to calculate the corresponding C_{pu} for each sample.

1. Choose a distribution with known parameters, for example, Weibull $\alpha = 1.2$ and $\beta = 1$.
2. Find $X_{0.99865}$ and $X_{0.5}$ for this distribution using any statistical package.
3. Choose a target C_{pu} value, say $C_{pu} = 1.5$.
4. Use (17) to calculate U_t, which equals 6.867 for this example.
5. Next we compare between the three methods, Clements, Box-Cox and Burr using the next series of steps.
6. Simulate values from underlying distribution.
7. Use each method to estimate $X_{0.99865}$ and $X_{0.5}$.
8. For a target C_{pu} value, say 1.5, and corresponding U_t value, say 6.867, calculate the C_{pu} values using all three methods (similar to Table 1).

9. Compare these calculated C_{pu} values using standard statistical measures and graphs to decide which, among the three methods, leads to the most accurate estimate of the target C_{pu} value.

The main criteria used to compare between the three methods is to determine the precision and accuracy of process capability estimations. The best and most suitable method will have the mean of the estimated C_{pu} values closest to the target value (that is, greatest accuracy) and will have the smallest variability, measured by standard deviation of the estimated values [9] (that is, greatest precision).

3.2 Simulation runs

As discussed in Section 3.1, we generate 30 samples, each of size 100, from specific Weibull, Gamma and Lognormal distributions. After each simulation run, the necessary statistics, such as mean, standard deviation, median, skewness, kurtosis, upper and lower 0.135 percentiles were obtained. In this article, C_{pu} is used as comparison criterion. The capability index for the non-normal data should be compatible with that computed under normality assumption, given the same fraction of non-conforming parts [9]. The estimates for C_{pu} were determined using Burr, Clements and Box–Cox methods steps outlined in Section 3.1. The average value of all 30 estimated values and their standard deviations were calculated and presented in Tables 2–4.

To investigate the most suitable method for dealing with non-normality presented by Weibull, Gamma, and Lognormal distributions, we present box plots of estimated C_{pu} values using all three methods (Figures 1–3). Box plots are able to graphically display important features of the simulated C_{pu} values such as median, inter-quartile range and existence of outliers. These figures indicate that the means using Burr method is closest to their targeted C_{pu} values and the spread of the values is smaller than that using Clements

Table 2: The mean and standard deviation of $30 C_{pu}$ values with $n = 100$ (Weibull).

		Burr		Clements		Box-Cox	
C_{pu}	U_t	mean	Std	mean	Std	mean	Std
0.5	2.780	0.596	0.090	0.590	0.099	0.621	0.100
1.0	4.824	1.152	0.159	1.159	0.175	0.956	0.194
1.5	6.867	1.708	0.228	1.727	0.252	1.204	0.283
2.0	8.910	2.264	0.297	2.296	0.328	1.407	0.367

Table 3: The mean and standard deviation of $30 C_{pu}$ values with $n = 100$ (Gamma).

		Burr		Clements		Box-Cox	
C_{pu}	U_t	mean	Std	mean	Std	mean	Std
0.5	3.650	0.578	0.091	0.593	0.105	0.611	0.075
1.0	6.608	1.117	0.166	1.159	0.188	0.897	0.132
1.5	9.565	1.655	0.241	1.725	0.271	1.099	0.185
2.0	12.522	2.194	0.316	2.290	0.354	1.262	0.233

method, therefore indicating a better approximation. Box–Cox method gives comparable results for smaller target values.

3.3 Discussion

As mentioned in Section 3.1, the performance yardstick is to determine the accuracy and precision for a given sample size. To determine accuracy, we looked at the mean of the estimated C_{pu} values and for precision, we focused on the standard deviation of these values using all three methods. Looking

Table 4: The mean and standard deviation of $30C_{pu}$ values with $n = 100$ (Lognormal).

		Burr		Clements		Box-Cox	
C_{pu}	U_t	mean	Std	mean	Std	mean	Std
0.5	4.482	0.499	0.084	0.496	0.097	0.503	0.048
1.0	8.339	1.024	0.166	1.031	0.183	0.710	0.061
1.5	12.009	1.523	0.243	1.541	0.265	0.832	0.070
2.0	15.679	2.022	0.320	2.050	0.348	0.921	0.076

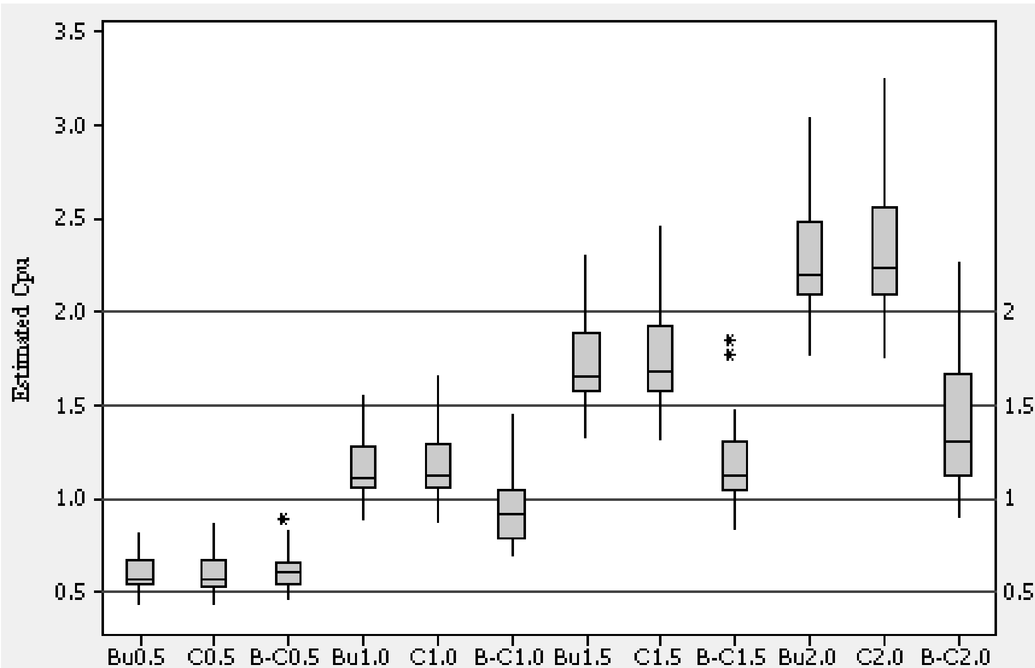

Figure 1: Box plot of estimated C_{pu} values with target $C_{pu} = 0.5, 1.0, 1.5$ and 2.0 for Weibull distribution.

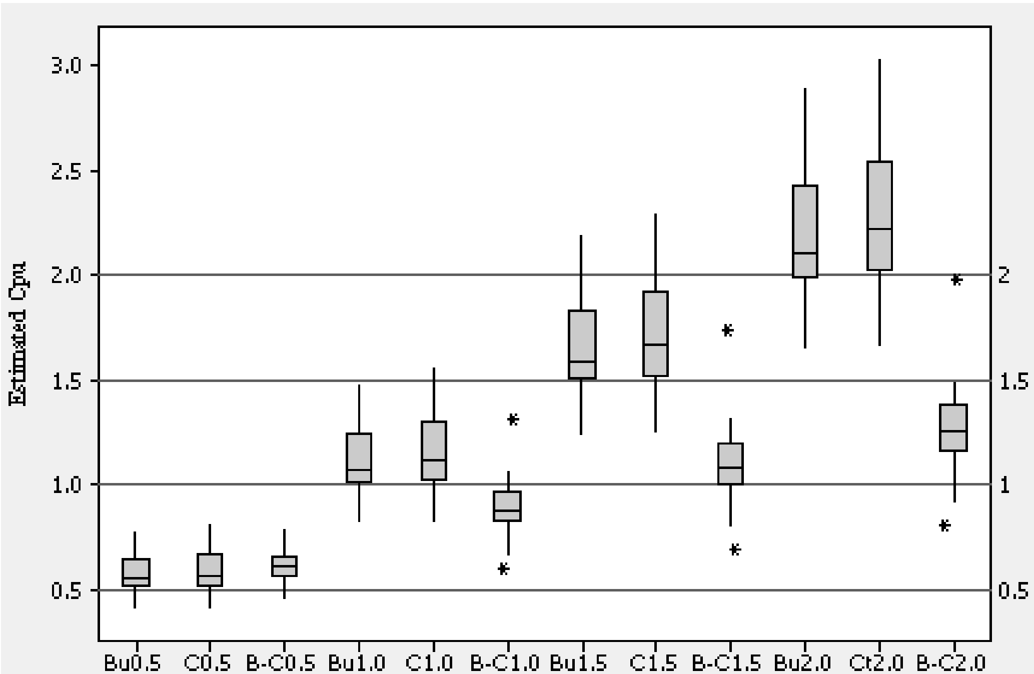

Figure 2: Box plot of estimated C_{pu} values with target $C_{pu} = 0.5, 1.0, 1.5$ and 2.0 for Gamma distribution.

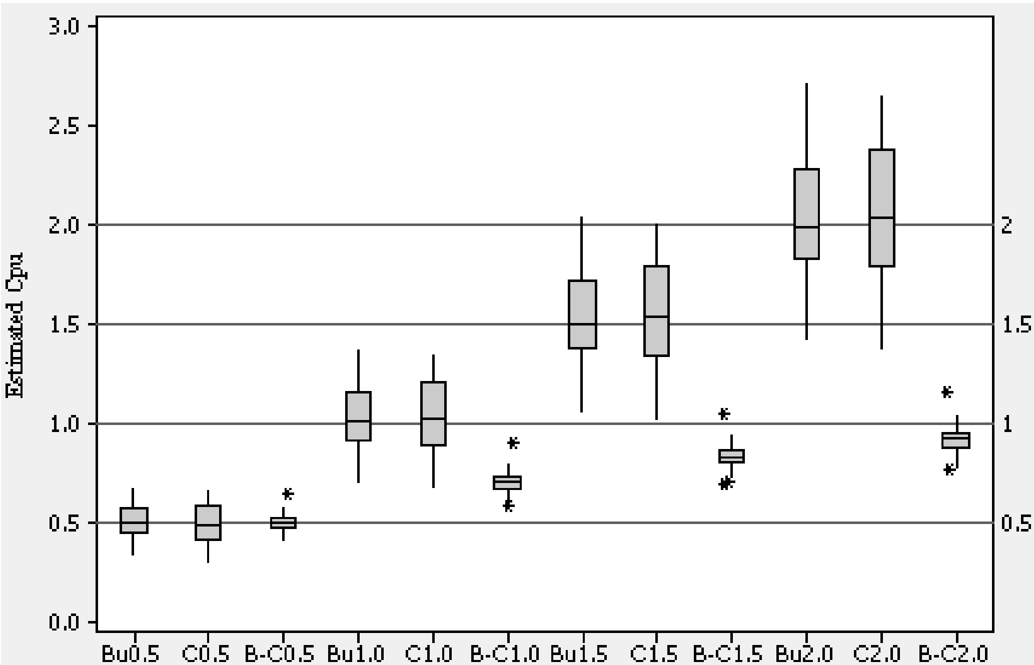

Figure 3: Box plot of estimated C_{pu} with target $C_{pu} = 0.5, 1.0, 1.5$ and 2.0 for Lognormal distribution.

at the results depicted in Tables 2–4, we conclude that

1. The Burr method is the one for which the mean of the estimated value deviates least from the targeted C_{pu} values.
2. The standard deviation of the estimated C_{pu} values using the Burr method is smaller than Clements method.
3. Box–Cox method does not yield results close to any targeted C_{pu} values except for smaller targeted values.

During our simulation exercises, we also observed that larger sample sizes yield better estimates for all methods. Therefore, sample size does have an impact on process capability estimate. It was also observed that larger targeted C_{pu} values led to slightly worse estimates for all methods.

4 Case studies

4.1 Example 1

The real data set, consisting of 30 independent samples each of size 50, is from a semiconductor manufacturing industry. The data are measurements of bonding area between two surfaces with upper specification $U_t = 24.13$. Initially, an $\bar{X}$-R chart was used to check whether or not the process is stable before further analysis of the experimental data. Figure 4 shows the histogram of the data. Using a Goodness of Fit Test, the data is best fitted by a Gamma distribution with $\hat{\alpha} = 2543.8$ and $\hat{\beta} = 0.00921$.

We used all three methods to estimate C_{pu}. The mean and standard deviation of the estimated C_{pu} values using each method is presented in Table 5. The actual C_{pu} value of this process, derived using equation (5)

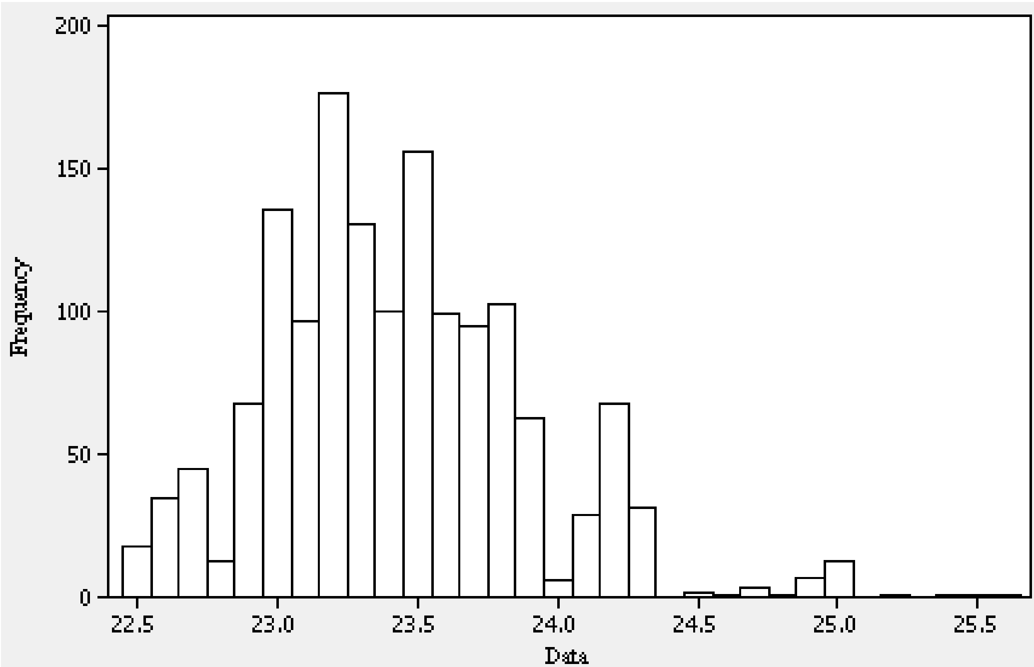

Figure 4: Histogram of data from a semiconductor industry.

and based on 1500 products, is 0.3775. The results in Table 5 show that the mean value obtained using Burr method is closest to the actual value.

4.2 Example 2

In the second case study, we present a capability analysis using a real set of data obtained from a computer manufacturing industry in Taiwan [15]. Again, the analysis is based on 30 independent samples, each of size 50. The data set has the one sided specification limit $U_t = 0.2\,\text{mm}$. Figure 5 shows the histogram of the data which shows that the underlying distribution is not normal and is right skewed. However, the data do not appear to be best fitted

Table 5: Process capability analysis results for semiconductor data.

Method	U_t	C_{pu} mean	C_{pu} Std
Clements	24.13	0.4368	0.0934
Burr	24.13	0.4207	0.0768
Box–Cox	24.13	0.4999	0.0608

Table 6: Process capability analysis results for computer manufacturing process.

Method	U_t	C_{pu} mean	C_{pu} Std
Clements	0.2 mm	2.0432	0.4354
Burr	0.2 mm	1.9251	0.3490
Box-Cox	0.2 mm	1.2431	0.2266

by any of the distributions we have used in this paper. All three methods have been applied to estimate process capability of this right skewed data. The estimated C_{pu} results are displayed in Table 6. The actual C_{pu} value of this process is 1.8954 and one of the reasons for selecting this example is to compare the three methods for a process where the capability index is greater than 1.5. The results in Table 6 again show that mean value obtained using the Burr method is closest to the actual value.

5 Conclusions

The main purpose of this article is to compare and contrast between three methods of obtaining process capability indices and determine which method is more capable in achieving higher accuracy in estimating these indices for

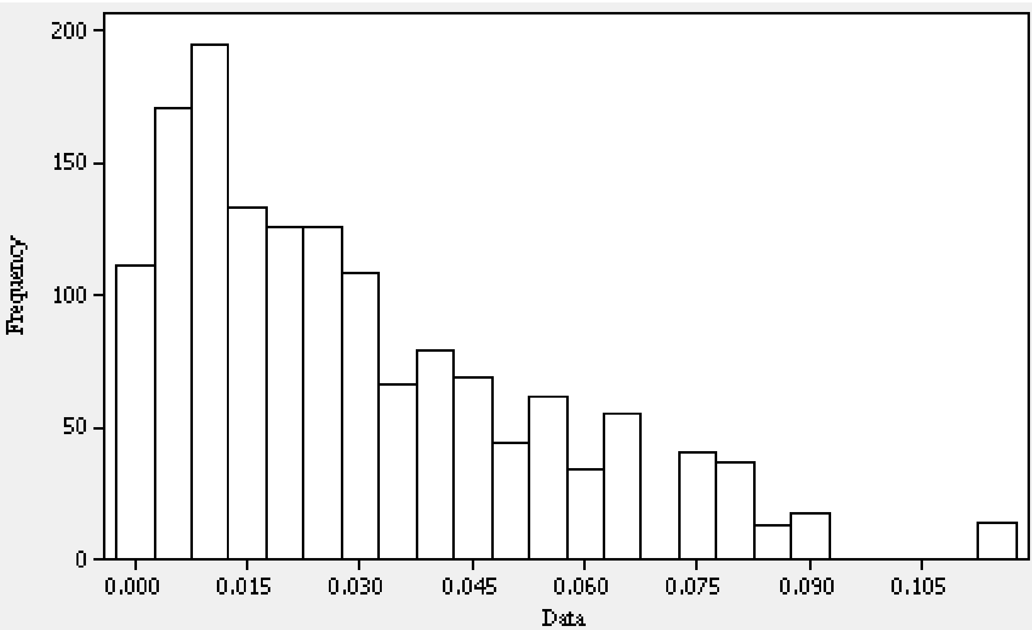

Figure 5: Histogram of measurement data from a computer manufacturing process.

non-normal quality characteristics data. Simulation study indicates that Burr's method generally provides better estimate of the process capability for non-normal data. Finally, two real examples from industry are presented. The results using these experimental data show that the estimated C_{pu} values obtained using Burr method are closest to the true values compare to other methods. In conclusion, Burr method is therefore deem to be superior to the other two methods for estimating the process capability indices for non-normal data. However, we strongly recommend further investigation of the Burr method for calculating PCIs for data whose underlying distributions show significant departures from normality.

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References

[1] I. W. Burr (1942). Cumulative frequency distribution. *Ann Math Stat* 13: 215–232. http://www.ams.org/mathscinet/pdf/6644.pdf C645, C647, C648

[2] I. W. Burr (1973). Parameters for a general system of distributions to match a grid of α_3 and α_4. *Commun Stat* 2:1–21. http://www.zentralblatt-math.org/zmath/search/?q=an:03413665&type=pdf&format=complete C648

[3] G. E. P. Box and D. R. Cox (1964). An analysis of transformation. *J Roy Stat Soc B* 26:211–252. http://www.jstor.org/view/00359246/di993152/99p02493/0 C645, C647

[4] J. A. Clements (1989). Process capability calculations for non-normal distributions. *Quality Progress* 22:95–100.

http://www.asq.org/qic/display-item/index.html?item=14059 C645, C646

[5] N. L. Johnson (1949). System of frequency curves generated by methods of translation. *Biometrika* 36:149–176. http://www.jstor.org/view/00063444/di992300/99p0220l/0 C645

[6] S. Kotz and C. R. Lovelace (1998). *Process capability indices in theory and practice*. Arnold, London. http://www.amazon.com/Process-Capability-Indices-Theory-Practice/dp/0340691778 C651

[7] S. Kotz and N. L. Johnson (1993). *Process capability indices*. New York: Chapman & Hall. http://www.amazon.com/Process-Capability-Indices-Samuel-Kotz/dp/041254380X C645

[8] L. A. R. Rivera, N. F. Hubele and F. D. Lawrence (1995). C_{pk} index estimation using data transformation. *Comput Ind Engng* 29: 55-58. http://www.ingentaconnect.com/content/els/03608352/1995/00000029/00000001/art00045 C649

[9] L. C. Tang, S. E. Than (1999). Computing process capability indices for non-normal data : a review and comparative study. *Qual Reliab Engng Int* 15: 339–353. doi:CCC 0748-8017/99/050339 C644, C647, C651, C653

[10] D. Montgomery (1996). *Introduction to Statistical Quality Control*. 5th edition. Wiley, New York. http://bcs.wiley.com/he-bcs/Books?action=index&bcsId=2077&itemId=0471656313 C644

[11] J. O'Connell and Q. Shao (2004). Further investigation on a new approach in analyzing extreme events. *CSIRO Mathematical and Information Sciences* Report No. 04/41. http://www.cmis.csiro.au/techreports/docs/x0000ihw.pdf C649

[12] Pei–Hsi Liu and Fei–Long Chen (2006). Process capability analysis of non-normal process data using the Burr XII distribution. *Int J Adv Manuf Technol* 27: 975–984. doi:10.1007/s 00170-004-2263-8 C646, C647, C648, C651

[13] S. Somerville and D. Montgomery (1996). Process capability indices and non-normal distributions. *Quality Engineering* 19(2):305–316. doi:10.1080/08982119608919047 C645

[14] F. K. Wang (2006). Quality evaluation of a manufactured product with multiple characteristics. *Qual Relib Engng Int* 22: 225–236. `http://doi.wiley.com/10.1002/qre.712` C644

[15] H. H. Wu, J. S. Wang and T. L. Liu (1998). Discussions of the Clements-based process capability indices. In: *Proceedings of the 1998 CIIE National Conference* pp.561–566. doi:10.1007/s 00170-004-2263-8 C646, C659

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