Numerical study of currents generated in a lake, and verification by experiment

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Abstract

A finite element method was applied to equations of fluid dynamics to simulate currents generated in a lake. Numerical results were introduced into analysis of the motion of the float equipped with a GPS unit, and a numerical result was compared with an experimental result. Numerical results concerning currents can be used in analysis of sediment transport in a lake. They can also be used in study of water quality of a lake.

Contents

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1 Introduction

Currents generated in Kojima Lake are analyzed. This lake is located on the coast of the Seto Inland Sea, which separates the Shikoku Island and the main island of Japan. Kojima Bay is connected to the Seto Inland Sea, and a portion was isolated by a bank to become Kojima Lake, which is surrounded by two municipalities, Okayama and Tamano in the Okayama Prefecture. It is approximately $10\,\mathrm{km}^2$ in area. Its average depth ranges from approximately 1.8 m to 2.1 m. The primary water source for the Kojima Lake is the supply from two rivers, the Kurashiki River and the Sasagase River (Figure 1).

The bank isolating Kojima Lake has six gates, which are opened when necessary to discharge water to Kojima Bay in order to lower the lake's water level. In such an event, a current is generated in the Kojima Lake. On July 16, 2007, those gates were opened from approximately 7:00 to 8:40 AM (GMT). Figure 2 shows change of the water levels of the Kojima Lake during the period. The change of water level of the Kurashiki river, the Sasagase river, and the Kojima Lake and Kojima Bay are also shown. These data were introduced into a finite element analysis of momentum equations and a continuity equation. Note that the water level of the lake was decreased by approximately 0.3 m during the period. Figure 3 shows the geography of Kojima Lake in the entire region and in a region near the gates [6].

Numerical results are obtained by introducing data concerning the depth

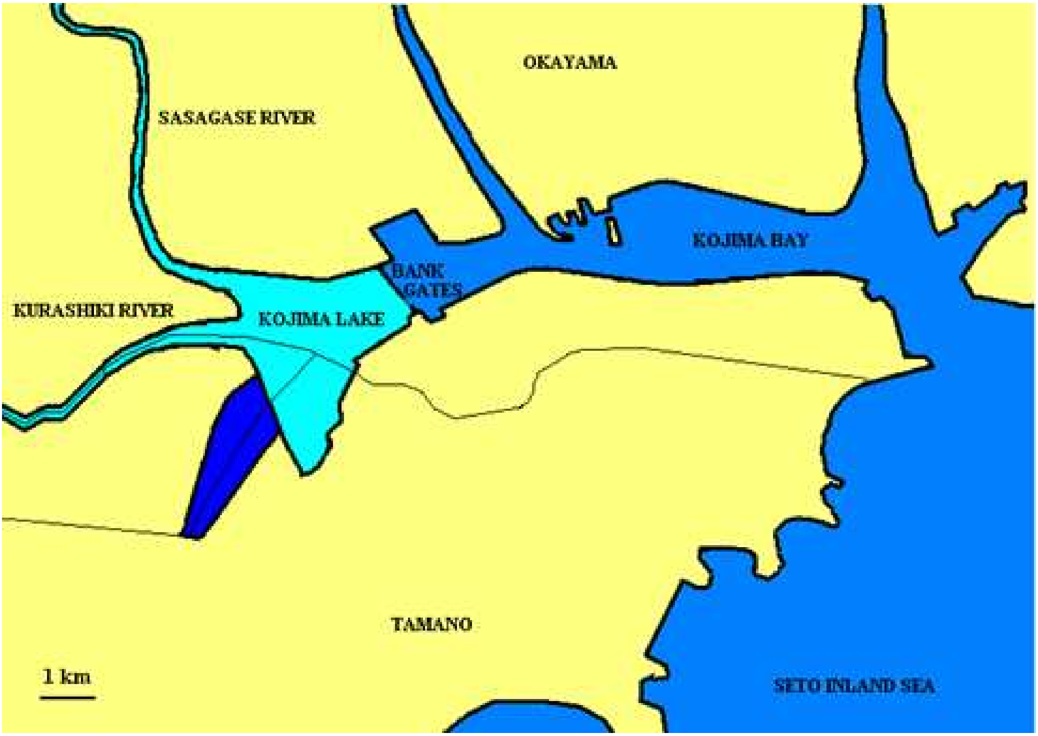

Figure 1: Geography of Kojima Lake and related areas. The map also indicates Kurashiki River, Sasagase River, the gates, Kojima Bay, Seto Inland Sea.

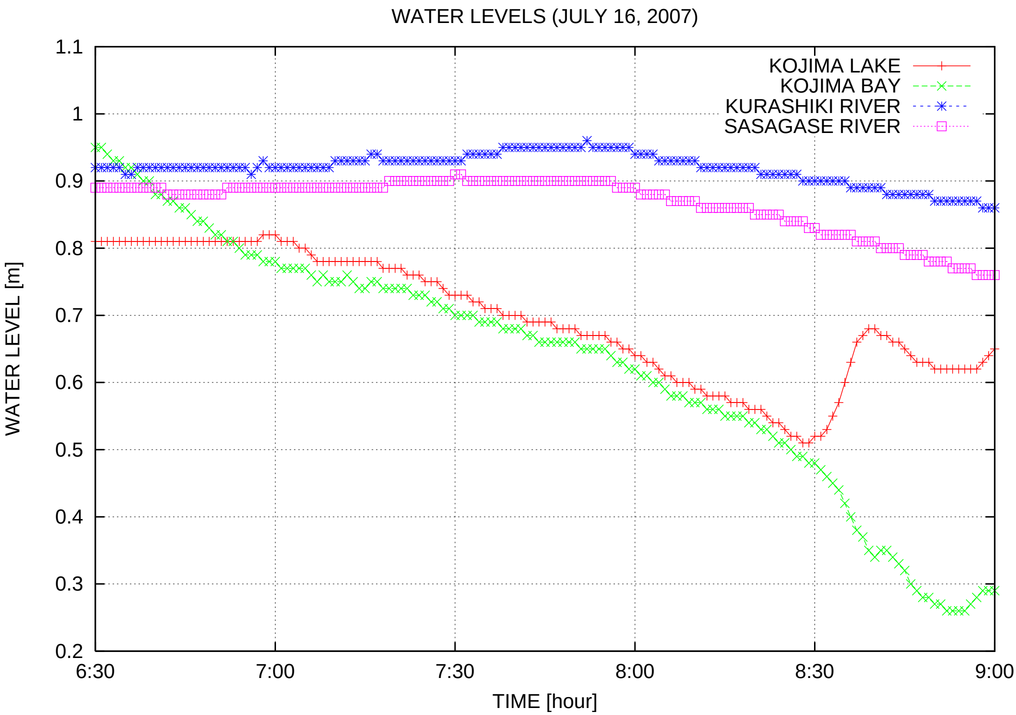

FIGURE 2: Change of water levels on July 16, 2007.

of the lake into finite element analysis of equations that govern the dynamics of the currents. Experimental and/or measuring techniques to utilize the global positioning system (GPS) in analyses of currents are also introduced. Numerical results of analysis of flow are tested against experiment using a float equipped with a GPS unit, called the GPS-float.

2 Analysis of flow in the Kojima Lake

A finite element method was applied to a system of partial differential equations (1) to analyze the current generated in Kojima Lake. These consist of momentum equations and a continuity equation [3, 4, 5]. Water flow in the lake is modeled by the shallow water equations,

$$
\begin{aligned}
\frac{\partial M}{\partial t} &= -g(h+\zeta)\frac{\partial \zeta}{\partial x} + A_h\left(\frac{\partial^2 M}{\partial x^2} + \frac{\partial^2 M}{\partial y^2}\right) - \frac{\tau_x}{\rho_0}\,,\\
\frac{\partial N}{\partial t} &= -g(h+\zeta)\frac{\partial \zeta}{\partial y} + A_h\left(\frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2}\right) - \frac{\tau_y}{\rho_0}\,,\\
\frac{\partial \zeta}{\partial t} &+ \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = 0\,.
\end{aligned}
\tag{1}
$$

where

$$
\tau_x = \frac{\rho_0\gamma^2\sqrt{M^2+N^2}}{(h+\zeta)^2}M\,, \quad \tau_y = \frac{\rho_0\gamma^2\sqrt{M^2+N^2}}{(h+\zeta)^2}N\,, \tag{2}
$$

the bottom friction coefficient $\gamma^2 = 0.0026$, τ_x and τ_y are shear stress terms, $h \equiv h(x,y)$ is the bottom topography, $z = \zeta(x,y)$ is the lake surface height, and M and N are obtained by integrating the x-component and the y-component of the velocity over the depth, respectively:

$$
M = \int_{-h}^{\zeta} u\,dz\,, \quad N = \int_{-h}^{\zeta} v\,dz\,. \tag{3}
$$

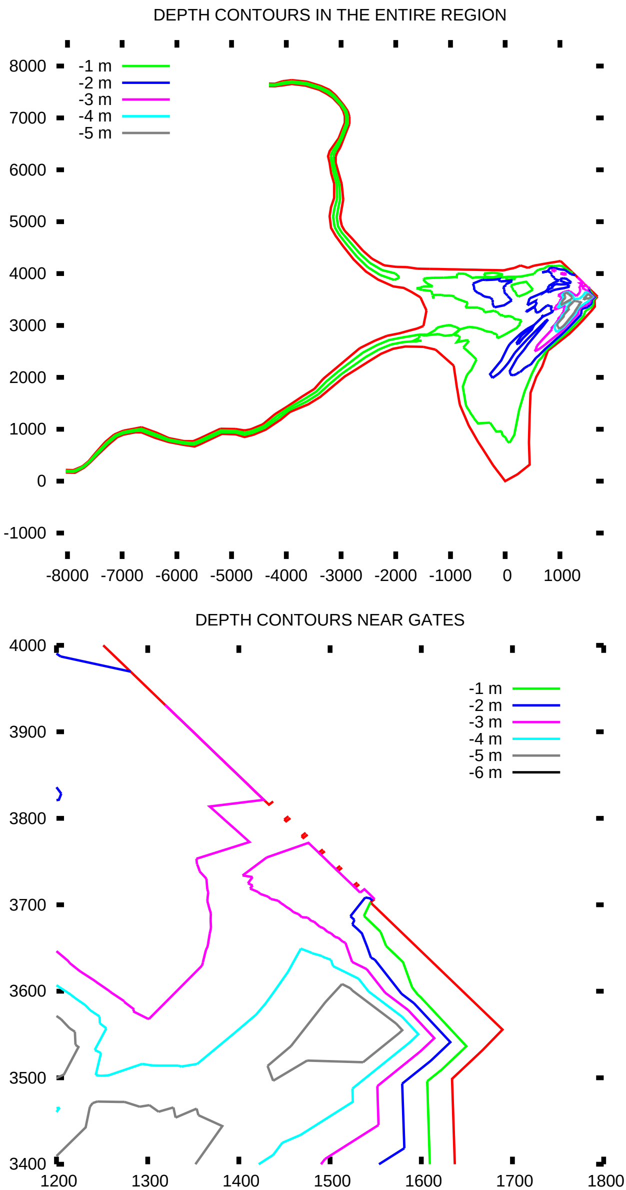

Figure 3: Geography of Kojima Lake. Depth contours in the entire region and in a region near the gates.

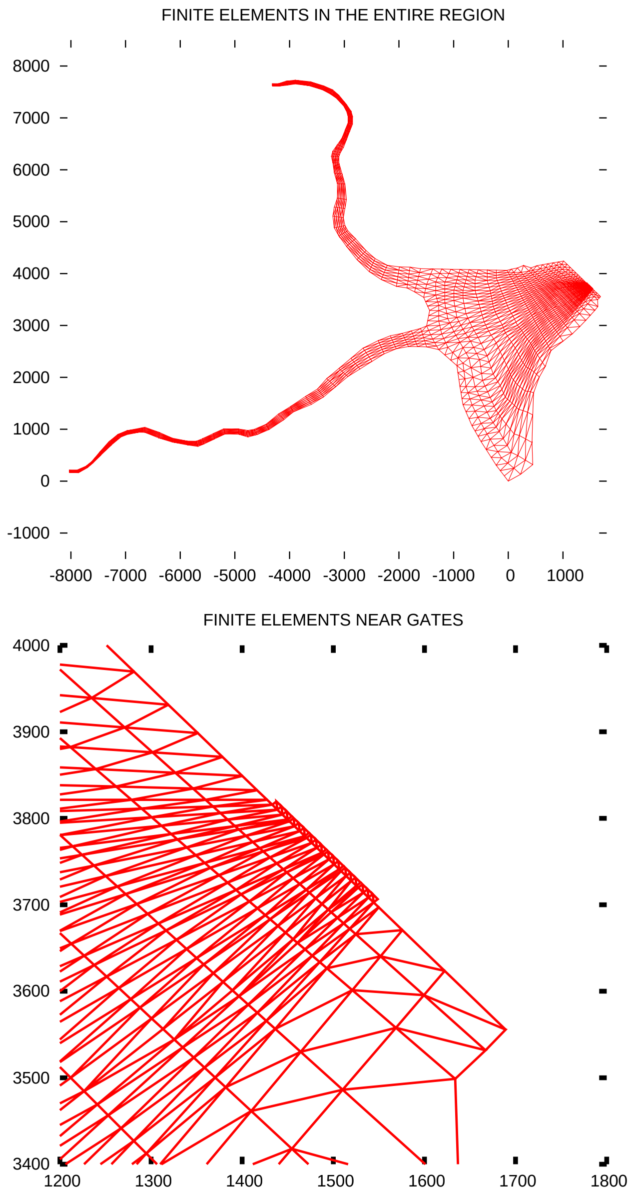

Figure 4: Finite element mesh in the entire region and in a region near the gates consisting of 2232 elements and 1246 nodes.

A finite element method was applied to the system (1) to simulate the flow generated in the Kojima Lake. Figure 4 shows the finite element mesh consisting of 2232 elements and 1246 nodes in the entire region and in a region near the gates. Figures 5 and 6 show numerically calculated velocity vectors at 40 and 80 minutes after the gates were opened on July 16, 2007, in the entire region and in a region near the gates.

3 GPS-float experiment and simulation

The GPS-float is a GPS-unit attached to a float with rectangular drag plates beneath the surface, and is designed to travel on the surface of the water with fluid resistance on the plates driving the float. While it travels on the surface, its temporal and positional data are transmitted via a wireless modem to be uploaded through a receiver into a PC (Figure 7). Data regarding the position of the GPS-float was used to verify the numerical results [2]. Table 1 shows the configuration of the GPS-float.

The GPS-float was used on July 16, 2007 on Kojima Lake. Its position is recorded, every second between 6.46 AM and 8.53 AM (GMT). During that period, the GPS-float traveled over 676 m with average velocity of approximately 0.089 m/s. Figure 8 shows the trajectory of the GPS-float. The numbers on the trajectories indicate the time elapsed after the gates were opened.

The GPS-float moves in response to fluid resistance on the pair of rectangular plates that are attached underneath the water surface. The fluid resistance is represented in terms of water velocities. Thus, the motion of the GPS-float is simulated by solving its momentum equations (7)–(8). Numerical results are tested against the experimental ones and good agreement was obtained. The vertical average of the velocity components $u\,(x,y,z,t)$, $v\,(x,y,z,t)$ should equal to $\bar{u}\,(x,y,t)$, $\bar{v}\,(x,y,t)$. They should also vanish at $z=-h\,(x,y)$. These assumptions lead to the following expression for

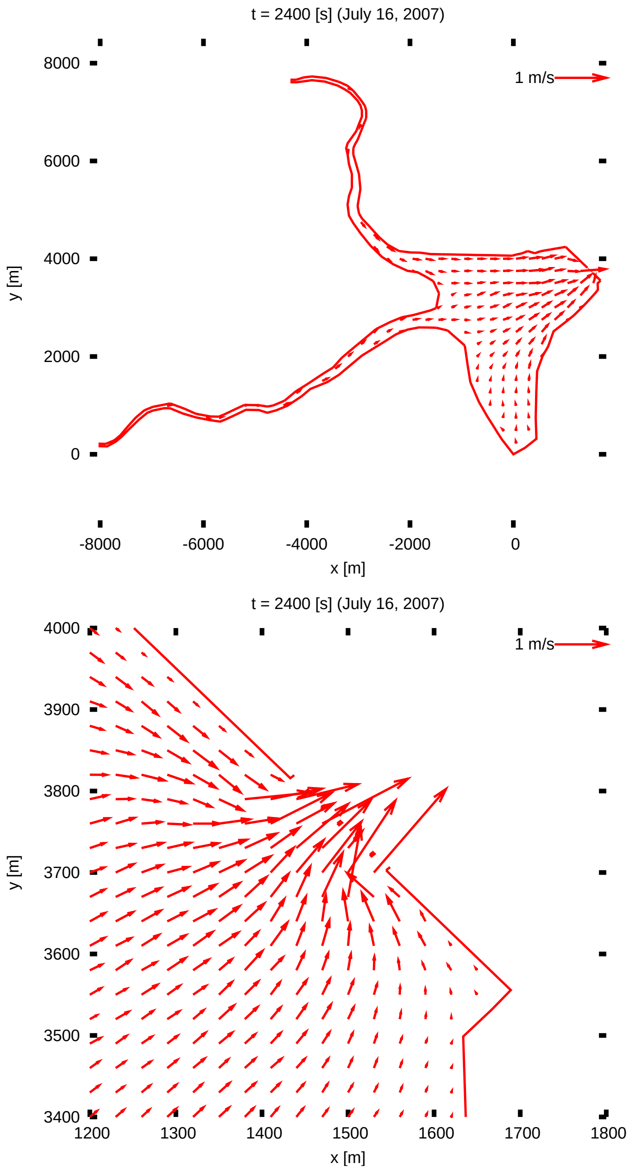

Figure 5: Velocity vectors in the entire region and in a region near the gates at time $t = 2400\,\mathrm{s}$ after the gates were opened on July 16, 2007.

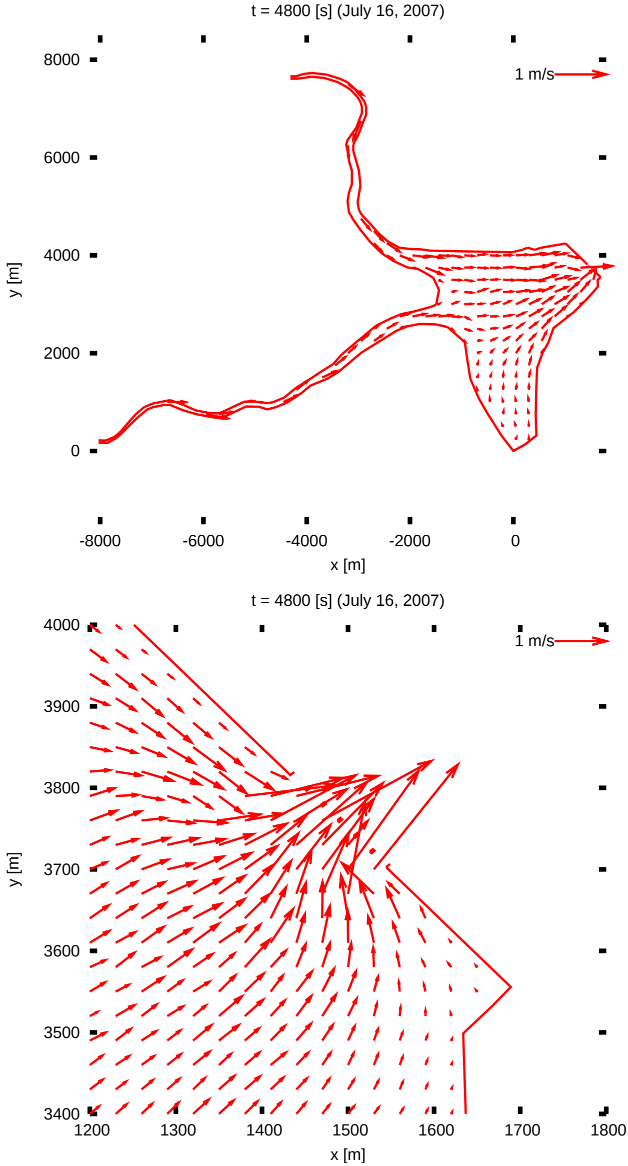

Figure 6: Velocity vectors in the entire region and in a region near the gates at time $t = 4800\,\mathrm{s}$ after the gates were opened on July 16, 2007.

Table 1: Configuration of the GPS-float [1]: VC, vinyl chloride; SS, stainless steel.

GPS unit	145(W) 51(H) 195(D)	
GPS antenna	155(D) 108(H)	
wireless modem(battery included)	72(W) 32(H) 133(D)	
float(first)	217(D) 223(H)	VC
float(second)	217(D) 264(H)	VC
GPS unit fitting	200(W) 223(D)	SS
wireless modem fitting	280(W) 410(H) 80(D)	SS
battery fitting	240(W) 102(H) 23(D)	SS
resistance board×4	424(W) 250(H)	SS
GPS-float main body(core)1	18(D) 395(H)	SS
GPS-float main body(core)2	21(D) 602(H)	SS
resistance board fitting	334(H) 95(W) 100(D)	SS
battery	182(W) 22(H) 60(D)	
whole	approx. 1600(H) 850(W) 850(D)	

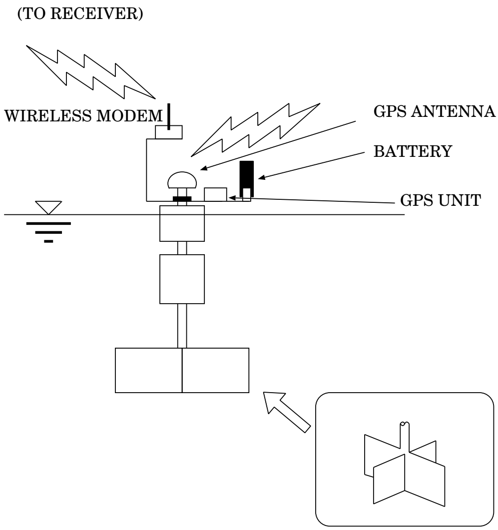

Figure 7: Illustration of the GPS-float.

$u(x,y,z,t)$ and $v(x,y,z,t)$ in terms of the averages $\bar{u}(x,y,t)$ and $\bar{v}(x,y,t)$:

$$\begin{aligned} u(x,y,z,t) &= (\gamma+1)\left(\frac{z+h(x,y)}{\zeta(x,y,t)+h(x,y)}\right)^{\gamma}\bar{u}(x,y,t)\,, \\ v(x,y,z,t) &= (\gamma+1)\left(\frac{z+h(x,y)}{\zeta(x,y,t)+h(x,y)}\right)^{\gamma}\bar{v}(x,y,t)\,. \end{aligned} \tag{4}$$

Here $\bar{u}$ and $\bar{v}$ are the vertically averaged velocity components

$$\bar{u} = \frac{1}{\zeta(x,y,t)+h(x,y)}M(x,y,t)\,, \quad \bar{v} = \frac{1}{\zeta(x,y,t)+h(x,y)}N(x,y,t)\,, \tag{5}$$

with fluxes M and N given by (3).

The driving force of the GPS-float is the fluid resistance on the rectangular plates underneath the surface. When a rectangular plate is fixed perpendicularly to a flow of fluid with uniform velocity U_∞, the fluid resistance D on the plate is proportional to the product of the dynamic pressure $\rho U_\infty^2/2$ and the area of the plate S, and is

$$D = \frac{C_D \rho S}{2} U_\infty^2 \,, \tag{6}$$

where ρ is the density of the fluid and the drag coefficient C_D varies from 1 to 18 [7]. When a plate moves in the fluid, and its position in the xy plane at time t is $(x(t), y(t))$, the magnitude of the fluid resistance should be proportional to the square of the magnitude of relative velocity $(u - \dot{x}, v - \dot{y})$, and its direction should be the direction of the relative velocity. Here (u, v) denotes the velocity of the fluid at time t and coordinate $(x(t), y(t))$ [8]. Thus we obtain the following momentum equations for the GPS-float:

$$m\ddot{x} = \frac{C_D S \rho_0}{2}(u - \dot{x})\sqrt{(u-\dot{x})^2 + (v-\dot{y})^2} \,, \tag{7}$$

$$m\ddot{y} = \frac{C_D S \rho_0}{2}(v - \dot{y})\sqrt{(u-\dot{x})^2 + (v-\dot{y})^2} \,. \tag{8}$$

We solved the above system to simulate the motion of GPS-float on a part of its trajectory with the following parameters: $\rho_0 = 1000\,\mathrm{kg/m^3}$ is the density of water, $C_D = 1.15$ is the non-dimensional drag coefficient, $m = 17.6\,\mathrm{kg}$ is the mass of the GPS-float, $S = 0.42\,\mathrm{m}^2$ is the surface area of the rectangular plates. Here we set $z = \zeta(x, y, t) - 0.8\,\mathrm{m}$ according to the position of the rectangular plates attached to the GPS-float. Figure 8 shows the numerical result and the experimental result. Table 2 shows the agreement between the numerical result and the experimental result.

4 Discussion

Whereas the vertically averaged velocity is generally sufficient to describe the flow in a shallow lake like Kojima Lake, it is important to analyze its vertical distribution to investigate problems such as sediment transport and water

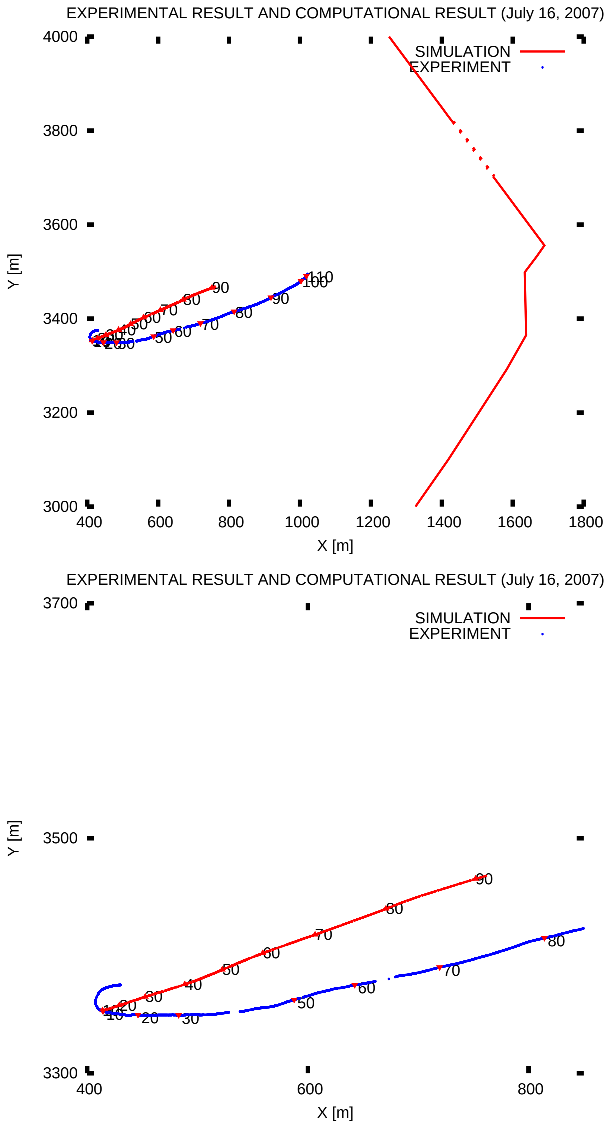

FIGURE 8: Trajectory of the GPS-float with elapsed time in minutes shown along side the path.

Table 2: Experimental and numerical results.

	Distance traveled by the GPS-float	
Elapsed time [min]	Experiment [m]	Simulation [m]
10–20	19.999758	16.452704
20–30	21.618499	28.226249
30–40	32.169115	43.770109
40–50	38.958587	38.715502
50–60	52.205503	48.085814
60–70	65.835841	66.588965
70–80	57.623421	68.364468
80–90	101.154554	86.909060

quality. Here we introduced a method to construct the vertical distribution of the horizontal velocity from the vertically averaged velocity.

We assumed the vertical distribution of the horizontal velocity is given in terms of the vertically averaged velocity. Solving equation (1) numerically for the vertically averaged velocity and using the result to solve the momentum equations (7)–(8) of the GPS-float, we simulated the motion of the GPS-float. The agreement between the experimental result and the numerical result was satisfactory, which supports our assumptions on the vertical distribution of the horizontal velocity.

In general, when an object is placed in a uniform flow of fluid, the drag exerted on the object by the fluid is proportional to the product of the density of the fluid, the square of the velocity of the fluid, and the cross sectional area of the object [7]. The proportion of the density of air to the water density is more or less one to one thousand, and the cross sectional area of the part of the float above the surface is smaller than the cross sectional area of the part underneath the water. According to the Japan Meteorological Agency,[1] the

[1] http://www.jma.go.jp/jma/index.html, accessed on January 26, 2008.

wind velocity recorded at every hour at three points surrounding the Kojima Lake was less than 3 m/s during 6:00 AM–9:00 AM (GMT). On the other hand Figure 5 indicates that the velocity of the water was greater than 0.1 m/s in the area where the GPS-float traveled. If the velocity of the wind was 3 m/s and the velocity of the water was 0.1 m/s, the drag on the float exerted by the water is more than ten times greater than the drag exerted by the wind, and the effects of the wind on the float was negligible. However, a wind-driven flow seemed to be formed before the gates were opened. That may be the reason that the flow traveled away from the gates for approximately the first ten minutes or less.

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