An asymptotic analysis for the flow between deformable rotating rolls

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Abstract

A simple case of the flow between deformable rolls is formulated as a multi-point boundary non-linear ODE's system. It is quite difficult to calculate solutions for this problem. Thus, the aim of this analysis is to find simple and effective solutions to this problem which are important for the understanding of the process. In this paper, asymptotic methods have been applied at the flow inlet where the flux through the gap is determined.

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Contents

1 Introduction

Many industrial processes involve flow between deformable rotating rolls. Such processes include coating flow and rolling of metal strip. A feature of this problem is the interaction between the solids and the flow contact with them.

An effective numerical solution for this problem is given in [3]. It is important to seek insight about such processes which would not be forthcoming from complicated analytical expressions or numerical solutions.

In this paper, the basic mathematical formulation for the flow between deformable rolls is outlined in Section 2. A reduced ODE's system for the flow through the inlet of the rolls is formulated and some asymptotic solutions are given in Section 3. A perturbation analysis using Airy functions is carried out in Section 4. Finally, in Section 5, the asymptotic solutions of Section 3 are used as the leading terms for the construction of more general solutions, and the Airy function approximations of Section 4 are applied to determine other terms.

2 Problem formulation

Consider the flow between two deformable rotating rolls as shown in Figure 1. Questions of interest include the minimum clearance between the two rolls

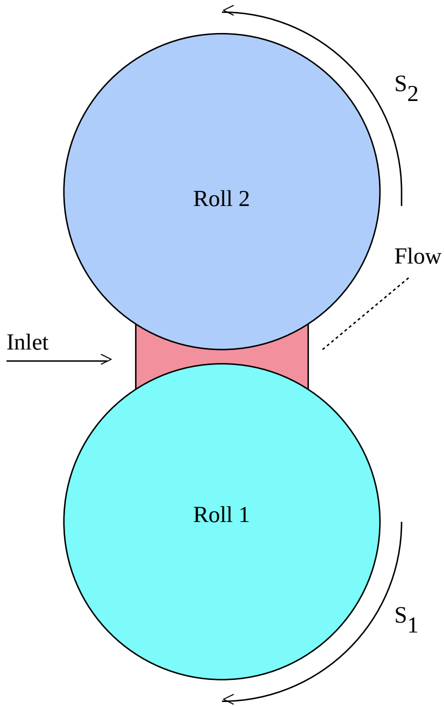

FIGURE 1: Schematic of flow between deformable rotating rolls

to achieve a given output thickness and the flux of fluid between them.

In the case when the fluid flow is isoviscous, Newtonian and incompressible (see [3]), the flow can be described by the Reynolds equation:

$$\frac{dp}{dx} = 12\mu S\left(\frac{h-h^*}{h^3}\right) \tag{1}$$

where μ is viscosity

$S=(S_1+S_2)/2$ with S_1 and S_2 are speeds of rolls

h is vertical separation between rolls

h^* is constant

The vertical separation between rolls depends on the geometry and the elastic deformation of the rolls. This gives,

$$h(x) = h_0(x)+v(x) \tag{2}$$

where $h_0(x) \doteq h_0(0)+\dfrac{x^2}{2R}$ (by parabolic approximation)

$1/R = 1/R_1 + 1/R_2$ (R_1 and R_2 are radii of rolls)

and $v(x)$ is the elastic deformation of rolls. From Hertz contact theory, see [2], $v(x)$ can be evaluated by a singular integral. However, in order to simplify the problem we assume that the quantity of shear stress is very small compared to the normal pressure. Then, the problem can be approximated by the flow between Winkler foundations. From Conway et al [1], we obtain

$$v(x) = \frac{tp(x)}{E^*} \tag{3}$$

where p is the pressure, E^* is the effective Young's modulus and t is the thickness of an equivalent Winkler foundation.

By carrying out an analysis similar to that given in [3], we let

$$\begin{aligned}
p(x) &= \int_{-\infty}^{x} \left(\frac{h-1}{h^3}\right) d\xi \quad \text{(scaled pressure distribution)} \\
z(x) &= \int_{-\infty}^{x} \xi \left(\frac{h-1}{h^3}\right) d\xi \quad \text{(scaled load distribution)} \\
w(x) &= \int_{-\infty}^{+\infty} \xi \left(\frac{h-1}{h^3}\right) d\xi \quad \text{(scaled total load)} \\
\lambda &:= \frac{12\mu St\sqrt{R}}{E^*(h^*)^{\frac{5}{2}}} \quad \text{and} \quad \epsilon := \frac{1}{\lambda^2}
\end{aligned}$$

and then scale

$$\begin{aligned}
x &\mapsto x\left(\frac{12\mu StR}{E^*(h^*)^2}\right), \quad h \mapsto hh^* \\
p &\mapsto p\lambda\,, \quad z \mapsto z\lambda\,, \quad w \mapsto w\lambda
\end{aligned}$$

to obtain the following multi-point boundary ODE's system:

$$\epsilon\frac{dh}{dx} = x + \left(\frac{h-1}{h^3}\right) \tag{4}$$

$$\frac{dp}{dx} = \frac{h-1}{h^3} \tag{5}$$

$$\frac{dz}{dx} = x\left(\frac{h-1}{h^3}\right) \tag{6}$$

$$\frac{dw}{dx} = 0 \tag{7}$$

Subject to boundary conditions

$$p(\pm\infty) = 0 \tag{8}$$

$$z(-\infty) = 0 \tag{9}$$

$$z(+\infty) + w(+\infty) = 0 \tag{10}$$

In practice, the value of ϵ is always very small ($10^{-9} < \epsilon < 10^{-4}$). From equation (4) we can prove that, on the left hand side of point $x = -\frac{4}{27}$, the vertical separation h can be approximated by a solution of the differential equation $x = \epsilon\frac{dh}{dx}$ while at the right hand side of this point h can be approximated by the implicit function $x + \left(\frac{h-1}{h^3}\right) = 0$. And, both approximations are invalid as x approaches this point. That gives a turning point at $(x, h) = (-\frac{4}{27}, \frac{3}{2})$.

3 Reduced ODE's system for the inlet problem

In this section, we derive an ODE's model for the flow, which can be solved using simple numerical techniques as well as asymptotic analysis. For example, the inlet geometry can be examined using a simple numerical method. Then the asymptotic solutions for the pressure distribution $p(x)$ and the total load W are obtained in simple forms.

From equations (4) and (5), the pressure distribution $p(x)$ is approximately symmetric when $\epsilon \mapsto 0$. Hence, we can simplify problem (4–7) by solving a half of the above rolling system from the inlet side. At the inlet of the rolls, $x < 0$. A change the variable $x := -x$ gives positive values for x. Thus, the ODE's system becomes

$$\epsilon \frac{dh}{dx} = x + \left(\frac{1-h}{h^3}\right) \tag{11}$$

$$\frac{dp}{dx} = \frac{1-h}{h^3} \tag{12}$$

$$\frac{dz}{dx} = -x\left(\frac{1-h}{h^3}\right) \tag{13}$$

The boundary conditions are

$$p(+\infty) = 0 \tag{14}$$

$$z(+\infty) = 0 \tag{15}$$

Note that another boundary condition is needed for this problem, and the turning point is now located at $(x, h) = (\frac{4}{27}, \frac{3}{2})$.

3.1 A sub-problem for h

Let $\bar{h}$ denote the asymptotic expansion about $h(0)$. Clearly, $\bar{h}$ is a function of ϵ at $x = 0$. From (11), we can derive the following perturbed iterative technique:

$$h_{i+1} = 1 - (\epsilon {h_i}' - x){h_i}^3$$

By using this technique with the initial value $h_o = 1$ when $\epsilon = 0$ and taking $h'_o = O(1)$, we obtain after three steps the following approximation at $x = 0$:

$$\bar{h}(\epsilon) \approx 1 - \epsilon + 6\epsilon^2 - 12\epsilon^3$$

This can be used as the additional boundary condition for the ODE's system (11–13). In this way, we have derived a simple initial value problem for h:

$$\epsilon \frac{dh}{dx} = x + \left(\frac{1-h}{h^3}\right) \quad \text{with} \quad h(0) \approx 1 - \epsilon + 6\epsilon^2 - 12\epsilon^3 \tag{16}$$

This problem can be easily solved numerically.

3.2 Pressure distribution (p)

Using (16), equation (12) can be written as:

$$p(x) \quad = \quad \int \frac{1-h}{h^3} dx = \int (\epsilon h' - x) dx \quad \text{by (11).}$$

Thus,

$$p(x) = \epsilon h - \frac{x^2}{2} + P_m \qquad (17)$$

where P_m is determined by the boundary condition $p(+\infty) = 0$.

3.3 Asymptotic solution for pressure

In the extreme case, as $\epsilon \longrightarrow 0$, equations (11) and (12) give $p = 0$ at $x = \frac{4}{27}$

$$\begin{aligned} \Rightarrow \bar{P}_m - \frac{1}{2}(\frac{4}{27})^2 &= 0 \quad \text{as} \quad \epsilon \longrightarrow 0 \\ \Rightarrow \bar{P}_m &= \frac{8}{729} \end{aligned}$$

The asymptotic value of the maximum pressure:

$$\bar{P}_m = \frac{8}{729} \quad \text{as} \quad \epsilon \longrightarrow 0$$

Thus, the asymptotic solution of pressure (as $\epsilon \longrightarrow 0$):

$$\bar{p}(x) = \begin{cases} \frac{8}{729} - \frac{x^2}{2}\,, & -\frac{4}{27} \leq x \leq \frac{4}{27} \\ 0\,, & \text{otherwise} \end{cases} \qquad (18)$$

3.4 Load distribution (z)

Similarly, equation (13) gives

$$\begin{aligned} z(x) &= -\int_x^{+\infty} \xi \frac{1-h}{h^3} d\xi \\ &= -\int_0^{+\infty} \xi \frac{1-h}{h^3} d\xi + \int_0^x \xi \frac{1-h}{h^3} d\xi \\ &= z_o + \int_0^x \xi \frac{1-h}{h^3} d\xi \quad \text{by setting} \quad z_o = -\int_0^{+\infty} \xi \frac{1-h}{h^3} d\xi \\ &= z_o + x\int_0^x \frac{1-h}{h^3} d\xi - \int_0^x \int_0^\xi \frac{1-h}{h^3} d\xi d\eta \quad \text{via integration by parts.} \end{aligned}$$

Therefore,

$$\begin{aligned} z(x) &= z_o + x(\epsilon h + P_m - \frac{x^2}{2}) - \int_0^x (\epsilon h + P_m - \frac{x^2}{2}) d\xi \\ &= z_o + x\epsilon h + xP_m - \frac{x^3}{2} - \epsilon \int_0^x h d\xi - xP_m + \frac{x^3}{6} \\ &= z_o - \frac{x^3}{3} + \epsilon(xh - \int_0^x h d\xi) \end{aligned}$$

Thus,

$$z(x) = z_o - \frac{x^3}{3} + \epsilon \left(xh - \int_0^x h d\xi \right) \tag{19}$$

where $z_o = -\int_0^{+\infty} \xi \frac{1-h}{h^3} d\xi$.

3.5 Asymptotic solution for z and total load (w)

As $\epsilon \to 0$:

$$\bar{z}_o = \int_0^{\frac{4}{27}} \bar{p}(x)dx = \frac{2^6}{3^{10}}$$

Hence, the asymptotic solution of z is:

$$\bar{z}(x) = \begin{cases} 0\,, & x < -\frac{4}{27} \\ \frac{2^6}{3^{10}} + \frac{x^3}{3}\,, & -\frac{4}{27} \leq x \leq \frac{4}{27} \\ \frac{2^7}{3^{10}}\,, & x > \frac{4}{27} \end{cases} \tag{20}$$

Recall that the pressure distribution for the full problem is asymptotically symmetric. Thus, the asymptotic value of the total load:

$$\bar{w} \quad = \quad \frac{2^7}{3^{10}} \quad \text{as} \quad \epsilon \to 0 \quad \text{by } \bar{w} = 2\int_0^{\frac{4}{27}} \bar{p}(x)dx \tag{21}$$

4 Perturbation analysis using Airy functions

In this section, we outline a simple Airy functions model for the inlet geometry (h) and compare this result with the numerical solution of the second order Taylor expansion for h. We also carried out a higher order approximation for h based on the Airy functions.

It is also noted that we now restore the sign of variable x as in the full ODE's system (4–7). Let $F(h) = \frac{4}{27} + h^{-3} - h^{-2}$. Equation (4) can be written as $\epsilon\frac{dh}{dx} = x + \frac{4}{27} - F(h)$ Taylor expansion of $F(h)$ about $h = \frac{3}{2}$ gives:

$$F(h) \approx \frac{16}{81}(h - \frac{3}{2})^2 - \frac{256}{729}(h - \frac{3}{2})^3 + \frac{320}{729}(h - \frac{3}{2})^4 - \frac{1024}{2187}(h - \frac{3}{2})^5 \cdots$$

Note that $F = F' = 0$ at $h = \frac{3}{2}$.
At the turning point, let $X = x + \frac{4}{27}$ and $H = h - \frac{3}{2}$, we obtain

$$\epsilon\frac{dH}{dX} \approx X - \frac{16}{81}H^2 + \frac{256}{729}H^3 - \frac{320}{729}H^4 + \frac{1024}{2187}H^5$$

Now, scale

$$X \rightarrow X\left(\frac{9}{4}\epsilon\right)^{\frac{2}{3}} \tag{22}$$

$$H \rightarrow H\epsilon^{\frac{1}{3}}\left(\frac{9}{4}\right)^{\frac{4}{3}} \tag{23}$$

$$\epsilon \rightarrow \epsilon\left(\frac{4}{9}\right) \tag{24}$$

$$\Rightarrow \quad \frac{dH}{dX} \approx X - H^2 + 4\epsilon H^3 - \frac{45}{4}\epsilon^{\frac{4}{3}}H^4 + 27\epsilon^{\frac{5}{3}}H^5 \tag{25}$$

4.1 A simple Airy functions model

Taking the first two terms of the above expansion, we have

$$H' = X - H^2$$

Let $H = \frac{Y'}{Y}$. Hence $H^2 = (Y')^2/Y^2$, and

$$\begin{aligned} H' &= \frac{Y''}{Y} - \frac{(Y')^2}{Y^2} \\ H' &= X - H^2 \\ &= X - \frac{(Y')^2}{Y^2} \\ \Rightarrow \quad Y'' &= XY \end{aligned}$$

The general solution is

$$Y(X) = C_0 A_i(X) + C_1 B_i(X)$$

where

$$\begin{aligned} Ai(X) &= \left(\frac{1}{3^{\frac{2}{3}}\pi}\right) \sum_{k=0}^{\infty} \frac{\Gamma(\frac{k+1}{3}) \sin(\frac{2\pi}{3}(k+1))}{k!} (3^{\frac{1}{3}} X)^k \\ Bi(X) &= e^{\frac{i\pi}{6}} Ai(Xe^{\frac{2i\pi}{3}}) + e^{-\frac{i\pi}{6}} Ai(Xe^{-\frac{2i\pi}{3}}) \end{aligned}$$

$\Rightarrow$

$$H = \frac{C_0 A_i'(X) + C_1 B_i'(X)}{C_0 A_i(X) + C_1 B_i(X)}$$

Now, the asymptotic solution as $X \to +\infty$:

$$\begin{aligned}
A_i(X) &\approx \frac{1}{2}\frac{\exp(-\frac{2}{3}X^{\frac{3}{2}})}{X^{\frac{1}{4}}\sqrt{\pi}} \\
B_i(X) &\approx \frac{\exp(\frac{2}{3}X^{\frac{3}{2}})}{X^{\frac{1}{4}}\sqrt{\pi}} \\
A_i'(X) &\approx -\frac{1}{2}\frac{\exp(-\frac{2}{3}X^{\frac{3}{2}})X^{\frac{1}{4}}}{\sqrt{\pi}} \\
B_i'(X) &\approx \frac{\exp(\frac{2}{3}X^{\frac{3}{2}})X^{\frac{1}{4}}}{\sqrt{\pi}}
\end{aligned}$$

If $C_1 \neq 0$ (and for any values of C_0):

$$H \to \frac{\lim_{X\to+\infty} B_i'(X)}{\lim_{X\to+\infty} B_i(X)} = \sqrt{X} \quad \text{as} \quad X \to +\infty$$

Otherwise, $C_1 = 0$. This requires $C_0 \neq 0$, we obtain

$$H = \frac{A_i'(X)}{A_i(X)} \to -\sqrt{X} \quad \text{as} \quad X \to +\infty$$

Let

$$\begin{aligned}
S_{in} &= \{(X, H) : H' > 0\} \\
S_{out} &= \{(X, H) : H' < 0\}
\end{aligned}$$

Since the outer solution $(X, H) \in S_{out}$ (H is decreasing)

$$\Rightarrow H_\infty = -\sqrt{X} \quad \text{is the stable branch for} \quad H$$

Thus, the simple model for the inlet geometry is:

$$H = \frac{A_i'(X)}{A_i(X)} \tag{26}$$

A graphical comparison of the models is given in Figure 2 where the following colour codes have been used:

- red curve is the plot of h corresponding to the simple Airy functions model;
- green dotted curve is the plot of h corresponding to the second order of Taylor expansion model;
- blue dashed curve is the result corresponding to the numerical solution for the sub-problem (16); and
- cyan curve is the result corresponding to the outer solution of equation (16).

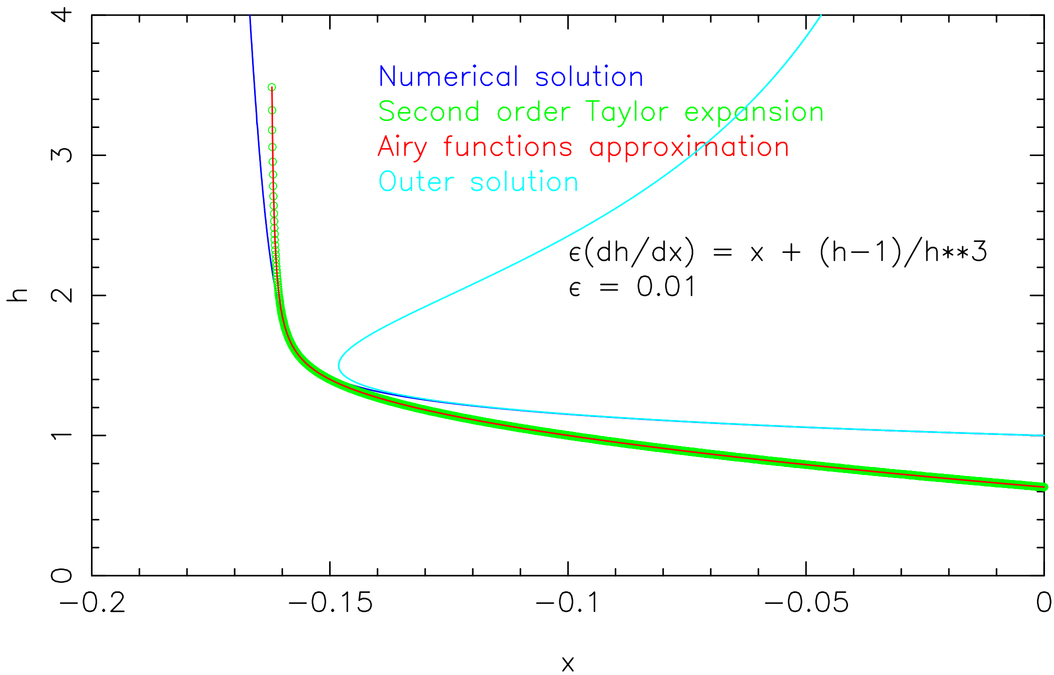

FIGURE 2: The inlet geometry of the models

4.2 Approximation using Airy functions

Now, assume

$$H = H_1 + \epsilon H_2 + \epsilon^{\frac{4}{3}} H_3 + \epsilon^{\frac{5}{3}} H_4$$

$$\Rightarrow \quad \frac{dH}{dX} = \frac{dH_1}{dX} + \epsilon \frac{dH_2}{dX} + \epsilon^{\frac{4}{3}} \frac{dH_3}{dX} + \epsilon^{\frac{5}{3}} \frac{dH_4}{dX}$$

From (25), we can write

$$\begin{aligned} \frac{dH}{dX} \approx\ & X - \left(H_1 + \epsilon H_2 + \epsilon^{\frac{4}{3}} H_3 + \epsilon^{\frac{5}{3}} H_4\right)^2 \\ & + 4\epsilon \left(H_1 + \epsilon H_2 + \epsilon^{\frac{4}{3}} H_3 + \epsilon^{\frac{5}{3}} H_4\right)^3 \\ & - \frac{45}{4}\epsilon^{\frac{4}{3}} \left(H_1 + \epsilon H_2 + \epsilon^{\frac{4}{3}} H_3 + \epsilon^{\frac{5}{3}} H_4\right)^4 \\ & + 27\epsilon^{\frac{5}{3}} \left(H_1 + \epsilon H_2 + \epsilon^{\frac{4}{3}} H_3 + \epsilon^{\frac{5}{3}} H_4\right)^5 \\ \approx\ & X - H_1^2 + \epsilon\left(4H_1^3 - 2H_1H_2\right) - \epsilon^{\frac{4}{3}} \left(\frac{45}{4} H_1^4 + 2H_1H_3\right) \\ & + \epsilon^{\frac{5}{3}} \left(27H_1^5 - 2H_1H_4\right) - \epsilon^2 (\cdots) \cdots \end{aligned}$$

Thus,

$$\begin{aligned} \frac{dH_1}{dX} &\approx X - H_1^2 \\ \frac{dH_2}{dX} &\approx 4H_1^3 - 2H_1H_2 \end{aligned}$$

$$\frac{dH_3}{dX} \approx -\frac{45}{4}H_1^4 - 2H_1H_3$$
$$\frac{dH_4}{dX} \approx 27H_1^5 - 2H_1H_4$$

Similarly, we find the asymptotic solutions for H_1, H_2, H_3 and H_4 to use as the boundary conditions for the above ODE's system. The iterative technique gives:

$$H_1(X) \to -X^{\frac{1}{2}} \quad \text{as} \quad X \to \infty$$
$$H_2(X) \to 2X \quad \text{as} \quad X \to \infty$$
$$H_3(X) \to \frac{45}{8}X^{\frac{3}{2}} + \frac{135}{16} \quad \text{as} \quad X \to \infty$$
$$H_4(X) \to \frac{27}{2}X^2 + 27X^{\frac{1}{2}} \quad \text{as} \quad X \to \infty$$

Therefore, theoretical solutions of H_1, H_2, H_3 and H_4 can be written in terms of simple Airy functions:

$$H_1(X) \approx \frac{A_i'(X)}{A_i(X)} \tag{27}$$

$$H_2(X) \approx \frac{4}{A_i^2(X)}\int_\infty^X \frac{(A_i'(\xi))^3}{A_i(\xi)}d\xi \tag{28}$$

$$H_3(X) \approx \frac{45}{4A_i^2(\xi)}d\xi\int_\infty^X \frac{(A_i'(\xi))^4}{A_i^2(X)} \tag{29}$$

$$H_4(X) \approx \frac{27}{A_i^2(X)}\int_\infty^X \frac{(A_i'(\xi))^5}{A_i^3(\xi)}d\xi \tag{30}$$

Finally, the geometry for the inlet can be approximated by the following asymptotic expansion:

$$H(X) \approx H_1(X) + \epsilon H_2(X) + \epsilon^{\frac{4}{3}} H_3(X) + \epsilon^{\frac{5}{3}} H_4(X) \tag{31}$$

5 Asymptotic analysis

In this section, simple solutions for this problem are derived and the approximate results for the total load are obtained.

By using the above Airy functions approximation, asymptotic expansions at the neighbourhood of the turning point for x, h and p are given by:

$$x \approx \frac{4}{27} + x_1 \epsilon^{\frac{2}{3}} + \cdots \tag{32}$$

$$h \approx \frac{3}{2} + h_1 \epsilon^{\frac{1}{3}} + \cdots \tag{33}$$

$$p \approx p_1 \epsilon^{\frac{2}{3}} + p_2 \epsilon + \cdots \tag{34}$$

5.1 Asymptotic expansion of $p(0)$

From (18) and (34), the first three terms of the asymptotic expansion for $p(0)$ are expected to be:

$$p(0,\epsilon) \approx \frac{8}{729} + A\epsilon^{\frac{2}{3}} + B\epsilon + o(\epsilon) \tag{35}$$

where A and B are unknown constants. Further work is required to obtain the theoretical solution for these constants. Here, we use the least squares method to fit numerical solutions of this problem to obtain: $A \approx 0.631361$ and $B \approx 1.85549$.

5.2 Asymptotic expansions of pressure and total load

Pressure at:

$$p(x,\epsilon) \approx \begin{cases} p(0,\epsilon) - \frac{x^2}{2}, & -\sqrt{2p(0,\epsilon)} \le x \le \sqrt{2p(0,\epsilon)} \\ o(\epsilon)\,, & \text{otherwise} \end{cases} \tag{36}$$

Total load:

$$w(\epsilon) \approx 2\int_0^{\sqrt{2p(0,\epsilon)}} (p(0,\epsilon) - \frac{x^2}{2})dx$$

Therefore,

$$w(\epsilon) \approx \frac{2}{3}(2p(0,\epsilon))^{\frac{3}{2}} \tag{37}$$

5.3 Results

The results in Table 1 show that our simple approximate solutions are very well fit with the numerical solutions for small values of ϵ. However, in the case of large ϵ, we need to add higher order terms for ϵ into the expansion (35) to improve the accuracy for this approximation.

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Table 1: Some results for total load $w(\epsilon)$

λ	ϵ	Numerical Solution	Approximate Solution	Relative Error
25	3.51166E-05	2.39371E-03	2.39128E-03	1.01596E-03
50	8.77915E-06	2.25338E-03	2.25267E-03	3.11017E-04
75	3.90184E-06	2.21651E-03	2.21638E-03	5.62072E-05
100	2.19479E-06	2.20050E-03	2.20057E-03	3.31402E-05
125	1.40466E-06	2.19182E-03	2.19197E-03	6.73532E-05
150	9.75461E-07	2.18648E-03	2.18665E-03	8.04663E-05
175	7.16665E-07	2.18290E-03	2.18308E-03	8.45194E-05
200	5.48697E-07	2.18036E-03	2.18054E-03	8.39233E-05
225	4.33538E-07	2.17848E-03	2.17865E-03	8.16584E-05
250	3.51166E-07	2.17703E-03	2.17720E-03	7.84397E-05
275	2.90220E-07	2.17589E-03	2.17606E-03	7.47442E-05
300	2.43865E-07	2.17498E-03	2.17513E-03	7.12872E-05
325	2.07791E-07	2.17422E-03	2.17437E-03	6.79493E-05
350	1.79166E-07	2.17360E-03	2.17374E-03	6.44922E-05
375	1.56074E-07	2.17307E-03	2.17320E-03	6.13928E-05
400	1.37174E-07	2.17262E-03	2.17274E-03	5.84126E-05
425	1.21511E-07	2.17223E-03	2.17235E-03	5.57899E-05
450	1.08385E-07	2.17189E-03	2.17200E-03	5.32866E-05
475	9.72759E-08	2.17159E-03	2.17170E-03	5.09024E-05
500	8.77915E-08	2.17133E-03	2.17144E-03	4.86374E-05
525	7.96295E-08	2.17110E-03	2.17120E-03	4.67300E-05
550	7.25550E-08	2.17089E-03	2.17099E-03	4.48227E-05
575	6.63830E-08	2.17070E-03	2.17080E-03	4.29153E-05
600	6.09663E-08	2.17053E-03	2.17062E-03	4.12464E-05
625	5.61866E-08	2.17038E-03	2.17047E-03	3.98159E-05