

Accuracy issues of Monte-Carlo methods for valuing American options

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(Received 16 September 2001; revised 23 July 2002)

Abstract

In this paper we discuss accuracy issues of the Monte-Carlo method for valuing American options. Two major error sources are discussed: the discretization error of numerical methods for simulating stochastic models and the statistical error of finite samples. As the explicit Euler method is dominant in the extant literature of computational finance, it is strongly recommended to use numerical methods with higher convergence order to reduce the discretization error. In this paper we use the trapezoidal method for simulating the one-factor and two-factor models for commodity prices. For the Monte-Carlo method for valuing American options, variance reduction techniques are applied to reduce the statistical error.

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⁰See <http://anziamj.austms.org.au/V44/CTAC2001/Tian> for this article,
© Austral. Mathematical Soc. 2003. Published 1 April 2003. ISSN 1446-8735

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1 Introduction

The valuation and optimal exercise of American options is one of the most important and difficult problems in option pricing theory. This type of derivatives is found in all major financial markets, including equity, commodity, foreign exchange, insurance and energy. However, despite recent advances, the valuation of American options remains one of the most challenging problems in derivative finance. For financial models driven by one stochastic process, the finite difference method and binomial techniques are widely used by researchers and financial analysts. But these methods become impractical when considering multiple factor models which give better description of practical financial problems. In recent years, Monte-

Carlo (MC) methods have become increasingly attractive compared with other methods for valuing American options with multiple factor stochastic processes [1, 2, 7, 9].

The major advantages of the MC method are its simplicity and flexibility. It can be used to deal with stochastic models which are driven by multiple Wiener processes and/or by other stochastic processes such as the Poisson process. In addition, the standard error of the estimate, according to the Central Limit Theorem, is $\mathcal{O}(1/\sqrt{N})$, where N is the number of simulations. This implies that the convergence speed of the MC method depends on the number of simulations but is independent of the dimension of the problems. This is the dominant advantage of the MC method. In addition, modern computers are much faster and have larger storage. For example, using parallel computers can reduce computing time significantly.

The major drawback of the MC method is its slow rate of convergence $\mathcal{O}(1/\sqrt{N})$. A large number of simulations are needed to obtain suitable results. Numerical techniques for improving the convergence properties of the MC method have been developed in recent years. Boyle et al. [1] reviewed these techniques which have been applied to financial applications.

Compared with the widely discussed statistical error, introduced by the finite sampling, little has been done for reducing the discretization error which is introduced by numerical methods for simulating stochastic models. The explicit Euler method is dominant in the extant financial literature. The reason is partly because of the difficulties in designing efficient numerical methods for solving stochastic differential equations (SDEs). The numerical behaviour of the explicit Euler method is poor because of its low convergence order and stability properties. Although the accuracy of this method is improved by reducing the step size, the computational cost is very large. In addition, this approach may be also restricted by the

number of exercise dates in financial practice. For stochastic models which are stiff in the deterministic and/or stochastic components, it is imperative to use methods with both higher convergence order and better stability properties, such as methods found in [8, 11, 12].

In this paper we discuss accuracy issues of the MC method for valuing American options. In Section 2 we will discuss the discretization error of numerical methods. The principle of the least-square Monte-Carlo (LSM) method for valuing American options is presented in Section 3. Three variance reduction techniques are presented in Section 4. Numerical results are reported in Section 5 for valuing American options of the one-factor and two-factor models for commodity prices.

2 Discretization errors

In this section we use the one-factor model for commodity prices, introduced by Schwartz [10], to discuss the accuracy issue of numerical methods for simulating stochastic models. The two-factor model is discussed in Section 5. This one-factor model gives the spot price which follows a mean reverting type of process

$$dS = \alpha(\mu - \lambda - \ln S)S dt + \sigma S dW(t), \quad (1)$$

where $S(t)$ is the spot price at time t , α is the mean reversion rate which indicate the speed of adjustment of the spot price back towards its long term level μ , σ is the spot price volatility, λ is the market price of energy risk and $W(t)$ is the Wiener process.

Letting $x = \ln S$ and applying the Itô lemma to the one factor model (1), the log price x is characterised by the Ornstein-Uhlenbeck process

$$dx = \alpha(\hat{\mu} - x) dt + \sigma dW(t), \quad (2)$$

where $\hat{\mu} = \mu - \lambda - \sigma^2/2\alpha$.

With appropriate boundary conditions, future and forward prices at time t with maturity s are equal and given by

$$F(t, s) = \exp \left[e^{-\alpha(s-t)} \ln S + (1 - e^{-\alpha(s-t)})\hat{\mu} + \frac{\sigma^2}{4\alpha}(1 - e^{-2\alpha(s-t)}) \right]. \quad (3)$$

Clelow and Strickland [3, 5] gave formulas to calculate the analytic European call option value for the one-factor model (1). The price of a European put option under the same condition can be given by the put-call parity.

The time period $[0, T]$ is divided into L subintervals with step size $h = T/L$. Here T is the option maturity date. Applying the explicit Euler method [8] to (2), gives

$$x_{n+1} = x_n + \alpha h(\hat{\mu} - x_n) + \sigma \Delta W_n, \quad (4)$$

where $\Delta W_n = W(t_{n+1}) - W(t_n)$ is a Gaussian random variable and $\Delta W_n \sim N(0, \sqrt{h})$. The Euler scheme (4) is of strong order one for equations with additive noise.

In order to improve the accuracy of numerical option values, we can solve the SDE (2) with smaller step size, and/or use methods with higher convergence order. Applying the trapezoidal method [8] with strong order two for solving SDE (2), gives

$$x_{n+1} = x_n + \alpha h \left(\hat{\mu} - \frac{x_n + x_{n+1}}{2} \right) + \sigma \Delta W_n. \quad (5)$$

For the simulated spot price $S_i = \exp(x_i)$, the futures or forward price $F(T, s)$ can be evaluated by (3) at T . Supposing that N simulations are obtained, then the value of the call option is obtained

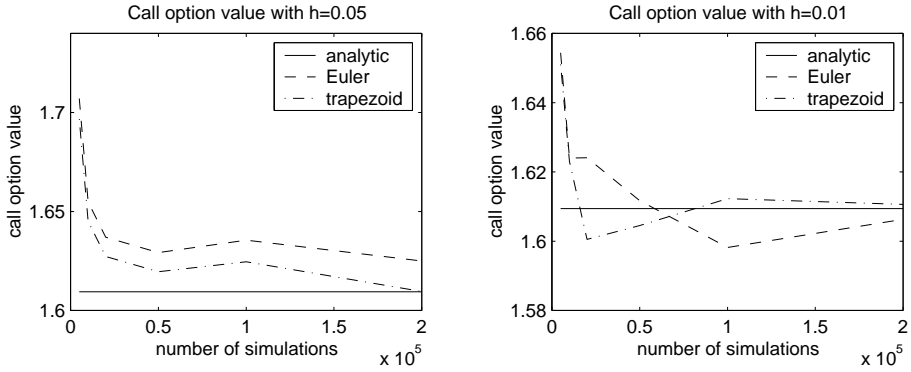


FIGURE 1: Numerical call option values by the Euler method and the trapezoidal method with $h = 0.05$ and $h = 0.01$.

by the discounted average of these simulated payoffs [5]:

$$\hat{E}_{\text{call}}(t) = P(0, T) \frac{1}{N} \sum_{i=1}^N \max(0, F_i(T, s) - K), \quad (6)$$

where $F_i(T, s)$ represents the forward price at T in the i th simulation, and $P(0, T) = \exp\left(-\int_0^T r(u) du\right)$ with the interest rate $r(u)$.

For this one-factor model, we use the data from [5] with $S_0 = \$26.90$, $\alpha = 0.472$, $\mu = 2.925$, $\sigma = 0.368$, $\lambda = 0$ and $K = 23.20$. We price a 6 month ($T = 0.5$) at-the-money (strike price equal to current futures price implying $F = K = 23.20$) option on a future contract with an original maturity of 1 year ($s = 1.0$). Interest rates are assumed to be constant at 10%. For these given data, the analytic European call option value is 1.6094.

Figure 1 gives call option values obtained by the explicit Euler method and the trapezoidal method. When the number of simulations N approaches infinity, the numerical option values do not

converge to the analytic value but converge to a different value with the discretization error. This phenomenon is best illustrated by the option values obtained by the explicit Euler method with $h = 0.05$. We obtain more accurate option values by the trapezoidal method and/or with a smaller step size $h = 0.01$. Certainly a smaller step size means better accuracy but the computing cost is very large.

Remark: the strong convergence order of a numerical method is more important when considering a stochastic model with multiplicative noises. For solving SDEs driven by multiplicative noises, the strong convergence order of the explicit Euler method is just half.

3 The least-square Monte-Carlo method

Under the risk-neutral measure, unlike the European call option price, given by

$$E_{\text{call}} = E \left[\exp \left(- \int_0^T r(\omega, u) du \right) \max\{S(T) - K, 0\} \right], \quad (7)$$

the American call option pricing problem is to find

$$A_{\text{call}} = \max_{\tau} E \left[\exp \left(- \int_0^{\tau} r(\omega, u) du \right) \max\{S(\tau) - K, 0\} \right] \quad (8)$$

over all stopping times $\tau \leq T$. Here $S(\tau)$ is the terminal stock price at time τ , K is the strike price, T is option maturity and $r(\omega, u)$ is the possible riskless interest rate associated with the realized path ω .

Here it is assumed that the American option can be only exercised at L discrete times $0 < t_1 \leq t_2 \leq \dots \leq t_{L-1} \leq t_L = T$. In practice, American options are continuously exercisable, and the option value is approximated by taking L to be sufficiently large.

At the final expiration date $t_L = T$, the option holder may exercise the option if it is in the money, or allow it to expire if it is out of the money. It is equivalent to a valuation problem for European-type securities (7). At exercise time t_i ($t_i < T$), the option holder must choose whether to exercise immediately or to continue the life of the option and revisit the exercise decision at the next exercise time t_{i+1} . At time t_i , the cash flow from immediate exercise is known and equals the value of immediate exercise. The cash flow from continuation is not known at time t_i . No-arbitrage valuation theory implies that the value of continuation $V(\omega, t_i)$ is given by taking the expectation of the remaining discounted cash flows $C(\omega, u; t_i, T)$ with respect to the risk-neutral pricing measure Q , namely

$$V(\omega, t_i) = E_Q \left[\sum_{j=i+1}^L \exp \left(- \int_{t_i}^{t_j} r(\omega, u) du \right) C(\omega, t_j; t_i, T) | \mathcal{F}_{t_i} \right]. \quad (9)$$

Here the expectation is taken conditional on the information set \mathcal{F}_{t_i} at time t_i . With this representation, the problem of optimal exercise reduces to comparing the immediate exercise value with the conditional expectation $V(\omega, t_i)$, and then exercising as soon as the immediate exercise value is not less than the conditional expectation.

For valuation of American options, Longstaff and Schwartz [9] introduced the LSM method to provide a pathwise approximation to the optimal stopping rule that maximizes the value of the American option. This method uses least squares to approximate the conditional expectation function $V(\omega, t_i)$ at t_i , $i = L - 1, \dots, 1$. We work backwards since the path of cash flows $C(\omega, u; t, T)$ generated

by the option is defined recursively; $C(\omega, u; t_i, T)$ can differ from $C(\omega, u; t_{i+1}, T)$ since it may be optimal to stop at time t_{i+1} , thereby changing all subsequent cash flows along a realized path ω . Here it is assumed that the unknown functional form of $V(\omega; t_i)$ can be represented as a linear combination of a countable set of \mathcal{F}_{t_i} -measurable basic functions. As an example in [9], Longstaff and Schwartz uses the powers of the state variable X as the basis functions, namely

$$V(\omega; t_i) = a_{i0} + a_{i1}X + a_{i2}X^2, \quad (10)$$

where a_{i0} , a_{i1} and a_{i2} are coefficients to be determined. Other functions, such as the Laguerre, Legendre, Chebyshev and Jacobi polynomials, can be also used as basis functions.

There are two major steps in the LSM method. With N simulations of the stochastic model, the first step is to estimate the coefficients of $V(\omega, t_i)$ in (10) by projecting or regressing the discounted values of $C(\omega, u; t_i, T)$ onto the basic function for the in-the-money paths at time t_i . Based on the conditional expectation function, the second step is to determine the early exercise decision at time t_i by comparing the immediate exercise value with the regression value $V(\omega; t_i)$ for each in-the-money path. Once the exercise decision is identified, the option cash flow paths $C(\omega, u; t_i, T)$ is approximated. The recursion proceeds by rolling back to time t_{i-1} and repeating the procedure until the exercise decisions at each exercise time along each path have been determined. Finally the American option is valued by

$$\hat{A}_{\text{call}} = \frac{1}{N} \sum_{k=1}^N \exp \left(- \int_0^{t_i^{(k)}} r(\omega, u) du \right) \max \{ S(t_i^{(k)}) - K, 0 \}, \quad (11)$$

where $t_i^{(k)}$ is the optimal valuation time for path k . If there is no optimal valuation time for path k , $\max \{ S(t_k^{(i)}) - K, 0 \} = 0$. For detailed description of the LSM method, see [9].

4 Variance reduction techniques

Suppose that we want to estimate a parameter θ by the i.i.d sequence $\{\hat{\theta}_i, i = 1, 2, \dots, N\}$, where each $\hat{\theta}_i$ has expectation θ and variance σ^2 . A natural estimator of θ based on these N replications is the sample mean $\hat{\theta}$ together with a sample variance estimation $\hat{\sigma}^2$:

$$\hat{\theta} = \frac{1}{N} \sum_{i=1}^N \hat{\theta}_i, \quad \hat{\sigma}^2 = \frac{1}{N-1} \sum_{i=1}^N (\hat{\theta}_i - \hat{\theta})^2.$$

By the Central Limit Theorem, for large N this sample mean is approximately normally distributed with mean θ and variance σ^2/N . The error in the estimator is proportional to σ/\sqrt{N} . Thus it is unwise to improve the accuracy of the estimator by increasing the number of replications because decreasing the error by a factor of 10 means increasing the number of replications by a factor of 100.

An alternative way to improve the accuracy is to use variance reduction techniques. There are several variance reduction techniques which are effective in financial application [1]. In this paper we apply three techniques, which are briefly introduced below, to the LSM method for valuing American options.

4.1 Antithetic variates

Suppose that $\{\Delta I_i\}$ are independent samples from the standard normal distribution $N(0, 1)$, then $\{-\Delta I_i\}$ are also independent samples from $N(0, 1)$. Denote $C_i, \hat{C}_i, (i = 1, \dots, N)$ as numerical security prices derived from $\{\Delta I_i\}$ and $\{-\Delta I_i\}$, respectively. The sampled security price is obtained by the standard MC method and the an-

tithetic variates method:

$$\hat{C}_{\text{MC}} = \frac{1}{N} \sum_{i=1}^N C_i; \quad \hat{C}_{\text{AV}} = \frac{1}{N} \sum_{i=1}^N \frac{C_i + \hat{C}_i}{2}.$$

A heuristic argument for preferring \hat{C}_{AV} notes that the random inputs in the antithetic variates method, $\{(I_n, -I_n)\}$, are more regularly distributed than a collection of $2n$ independent samples. In particular, the mean of these $2n$ samples is always 0.

The efficiency of the antithetic variates is measured by the variance reduction. As C_i and \hat{C}_i have the same variance,

$$\text{Var} \left[\frac{C_i + \hat{C}_i}{2} \right] = \frac{1}{2} (\text{Var}[C_i] + \text{Cov}[C_i, \hat{C}_i]),$$

thus $\text{Var}[\hat{C}_{\text{AV}}] \leq \text{Var}[\hat{C}_{\text{MC}}]$ if $\text{Cov}[C_i, \hat{C}_i] \leq \text{Var}[C_i]$. Notice that \hat{C}_{AV} uses twice as many replications as \hat{C}_{MC} . Thus the antithetic variate is effective if

$$2\text{Var}[\hat{C}_{\text{AV}}] \leq \text{Var}[\hat{C}_{\text{MC}}].$$

More detailed conditions can be found in [1] for the effectiveness of the antithetic variate technique.

4.2 Control variates

The idea in the control variates is “to use what we know”. Let \hat{C}_U be the numerical price of an option, obtained by N simulations, whose analytic value is not tractable. If we use the same simulations to calculate the price of another option \hat{C}_K whose analytic value C_K is known, the difference in the option values $\hat{C}_K - C_K$ is used to improve the accuracy of the estimated value \hat{C}_U :

$$\hat{C}_U^{cv} = \hat{C}_U + (C_K - \hat{C}_K).$$

In order to improve the efficiency of the control variates, we consider the following a family of unbiased estimators

$$\hat{C}_U^\beta = \hat{C}_U + \beta(C_K - \hat{C}_K) \quad (12)$$

with parameter β . The variance of the estimator is

$$\text{Var}[\hat{C}_U^\beta] = \text{Var}[\hat{C}_U] + \beta^2 \text{Var}[\hat{C}_K] - 2\beta \text{Cov}[\hat{C}_U, \hat{C}_K].$$

The estimator (12) will be effective by choosing the variance minimizing parameter β :

$$\beta_{\min} = \frac{\text{Cov}[\hat{C}_U, \hat{C}_K]}{\text{Var}[\hat{C}_K]}.$$

In practice we have two options for the control variates for the given N independent replications. The first is to use N_1 (typically $N_1 \ll N$) simulations for estimating β_{\min} and the other $N - N_1$ simulations for estimating \hat{C}_U^β . The disadvantage of this approach is the accuracy of β_{\min} . The second is to use these N simulations to estimate β_{\min} and the option value \hat{C}_U^β simultaneously. A more accurate estimator β_{\min} can be obtained but we must face the bias in the estimator \hat{C}_U^β . Notice that neither issue significantly limits the applicability of the method. The estimator of β_{\min} need not be very precise to achieve a reduction in variance, and the possible bias in the second implementation vanishes as N increases. The first approach will be used in this paper.

4.3 Moment matching methods

The idea of moment matching methods is to keep the generated random numbers satisfying the statistical properties of random variables. For a random variable Z with mean μ_Z and variance σ_Z^2 , the

samples Z_i ($i = 1, \dots, N$) normally do not satisfy the statistical properties, namely

$$\bar{Z} = \frac{1}{N} \sum_{i=1}^N Z_i \neq \mu_Z, \quad \bar{\sigma}_Z^2 = \frac{1}{N-1} \sum_{i=1}^N (Z_i - \bar{Z})^2 \neq \sigma_Z^2.$$

Thus by an appropriate transformation

$$\hat{Z}_i = (Z_i - \bar{Z}) \frac{\sigma_Z}{\sigma_Z} + \mu_Z, \quad (13)$$

the sample mean and the sample variance of \hat{Z}_i are μ_Z and σ_Z^2 .

Unlike the standard MC method, the confidence intervals of the true option value are not easy to obtain as the \hat{Z}_i ($i = 1, \dots, N$) are no longer independent. The numerical option values are biased estimators of the true option value. For most financial problems of practical interest, this bias is likely to be small. However, the bias can be arbitrarily large in extreme circumstances. The independence and bias in the moment matching methods make it difficult to quantify the improvement in general analytical terms.

5 Numerical results

In this section we first report numerical results for valuing European and American options based on the one-factor model for commodity prices (1). The stochastic differential equation (2) is solved with step size $h = 0.05$ by the trapezoidal method (5).

For the LSM method, the results are based on $N = 50000$ simulations. $N_1 = 25000$ pair simulations are used in the antithetic variates technique. For the control variates technique, $N_1 = 5000$ simulations are used for estimating parameter β_{\min} , and the other $N_2 =$

TABLE 1: Option values of one-factor model by MC methods

	Call option values			
	Euro.	Error	Amer.	S.E.
LSM	1.6058	0.0036	1.6258	0.0102
AVT	1.6136	0.0042	1.6275	0.0147
CVT	1.5980	0.0114	1.6252	0.0016
	Put option values			
	Euro.	Error	Amer.	S.E.
LSM	1.6179	0.0016	1.6292	0.0086
AVT	1.6106	0.0057	1.6291	0.0121
CVT	1.6183	0.0020	1.6282	0.0036

45000 simulations are used for valuing option values. The analytic European option values $E_{\text{call}} = 1.6094$ and $E_{\text{put}} = 1.6163$ are used as the control variates for the American options. For moment matching method, $M = 100$ batches of simulations are used. Each batch contains $N = 500, 1000, 5000$ or 10000 simulations.

Table 1 gives the European option values and errors, and American option values and the standard errors (S.E.) obtained by the LSM method, the antithetic variates (AVT) and control variates techniques (CVT). Table 2 gives option values and errors obtained by the moment matching method.

Next we discuss the two-factor model for commodity prices proposed by Schwartz [10]. In this model, the first factor is used to represent the spot price process S . Let $x = \ln(S)$, this process is

$$dx = \left(r - \frac{\sigma^2}{2} - \delta\right) dt + \sigma dW_1(t). \quad (14)$$

where r is the short term interest rate, δ is the convenience yield and σ is the volatility of the spot price S .

TABLE 2: Values of one-factor model by moment matching method

N	Call option values			
	Euro.	Error	Amer.	S.E.
500	1.6062	0.0032	1.6737	0.0053
1000	1.6017	0.0077	1.6507	0.0040
5000	1.6081	0.0013	1.6315	0.0017
10000	1.6113	0.0019	1.6282	0.0013
N	Put option values			
	Euro.	Error	Amer.	S.E.
500	1.6113	0.0050	1.6623	0.0046
1000	1.6081	0.0082	1.6466	0.0028
5000	1.6121	0.0042	1.6298	0.0013
10000	1.6146	0.0017	1.6287	0.0011

Instead of the constant convenience yield, the second factor is the instantaneous convenience yield of the spot energy and is assumed to follow the mean reverting process

$$d\delta = \alpha_\delta(\bar{\delta} - \delta) dt + \sigma_\delta dW_2(t). \quad (15)$$

where α_δ is the speed of adjustment, $\bar{\delta}$ is the long term mean of the convenience yield, and σ_δ represents the volatility of the convenience yield. With assumed constant interest rate r , futures and forward energy prices are equal. Schwartz [10] derived an expression for the futures price, and Clewlow and Strickland [4] gave formulas to calculate the analytic European option values.

Instead of the explicit Euler method used in [4, 5], we use the

trapezoidal method for solving the SDEs (14) and (15):

$$\begin{aligned}x_{n+1} &= x_n + \left(r - \frac{\sigma^2}{2} - \frac{\delta_n + \delta_{n+1}}{2} \right) h + \sigma \Delta W_{n1}. \\ \delta_{n+1} &= \delta_n + \alpha_\delta \left(\bar{\delta} - \frac{\delta_n + \delta_{n+1}}{2} \right) h + \sigma_\delta \Delta W_{n2}.\end{aligned}$$

The increments of the Brownian motions, ΔW_{n1} and ΔW_{n2} , are assumed to have correlation coefficient $h\rho_{S\delta}$. For the generated independent standard normal variates I_{n1} and I_{n2} , the increments of the Brownian motions can be represented by [5]

$$\Delta W_{n1} = \sqrt{\Delta t} I_{n1}, \quad \Delta W_{n2} = \sqrt{\Delta t} (\rho_{S\delta} I_{n1} + \sqrt{1 - \rho_{S\delta}^2} I_{n2}).$$

We can also use the Karhunen-Loeve expansion [6] to calculate the correlated random variates.

For the standard MC method, the results are based on $N = 50000$ simulations. For the antithetic variates technique, $N_1 = 25000$ pair simulations are used in valuation. In order to keep the correlation property, the antithetic variates are $\Delta \hat{W}_{n1} = -\Delta W_{n1}$ and $\Delta \hat{W}_{n2} = -\Delta W_{n2}$. For the control variates technique, $N_1 = 5000$ simulations are used for estimating parameter β_{\min} , and the other $N_2 = 45000$ simulations are used for valuing the European and American option values. The analytic European option values $E_{\text{call}} = 2.3701$ and $E_{\text{put}} = 2.3473$ are used as the control variates for the American option values. For moment matching method, $M = 100$ batches of simulations are used, each batch contains $N = 500, 1000, 5000$ or 10000 simulations.

Table 3 gives the European option values and errors, and American option values and the standard errors (S.E.) obtained by the LSM method, the antithetic variates (AVT) and control variates techniques (CVT). Table 4 gives option values and errors obtained by the moment matching method.

TABLE 3: Option values of two-factor model by MC methods

	Call option values			
	Euro.	Error	Amer.	S.E.
LSM	2.3757	0.0056	2.3947	0.0157
AVT	2.3692	0.0009	2.4080	0.0118
CVT	2.3696	0.0005	2.4061	9.3E-3
	Put option values			
	Euro.	Error	Amer.	S.E.
LSM	2.3358	0.0115	2.3622	0.0120
AVT	2.3492	0.0019	2.3758	6.2E-3
CVT	2.3297	0.0176	2.3712	5.0E-3

TABLE 4: Values of two-factor model by moment matching method

N	Call option values			
	Euro.	Error	Amer.	S.E.
500	2.3655	0.0046	2.4775	9.5E-3
1000	2.3692	0.0009	2.4469	6.6E-3
5000	2.3680	0.0021	2.4057	2.8E-3
10000	2.3620	0.0081	2.3983	1.9E-3
N	Put option values			
	Euro.	Error	Amer.	S.E.
500	2.3456	0.0017	2.4178	6.3E-3
1000	2.3482	0.0009	2.3943	3.9E-3
5000	2.3469	0.0004	2.3768	1.7E-3
10000	2.3438	0.0035	2.3737	1.3E-3

6 Conclusions

In this paper we have discussed two accuracy issues of the MC methods for valuing American options: the discretization error of numerical methods and statistical error of the MC methods. In order to reduce the discretization error, the trapezoidal method is applied to the one-factor and two-factor models for commodity prices. For valuing American options, variance reduction techniques are applied to reduce the statistical error. The following conclusions can be made from the discussion in this paper.

1. Figure 1 shows that the convergence order of numerical methods is very important to the accuracy of option values. Numerical methods with higher order and better stability properties are strongly recommended in financial application.
2. From results in Tables 1 and 3, the control variates technique is very effective for the commodity models, while the antithetic variates technique is not recommended, especially for the one-factor model.
3. Moment matching is effective for the LSM method. This approach is ideal for parallel computing. Unlike the valuation for European options, the accuracy of the valuation for American options heavily depends on the number of simulations in a trial. A large number of simulations in a trial is needed in order to get an acceptable results.
4. Numerical results in Tables 2 and 4 suggest that the simulated option values, obtained by the moment matching method, decrease monotonically to the true option values when the number of simulations in a trial become large. Thus the moment

matching method is a bias high approach for commodity models. As the LSM method is a bias low method [9], these properties can be used to get more accurate results.

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