Sludge formation in the activated sludge process with a sludge disintegration unit

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Abstract

The activated sludge process is one of the major aerobic processes used in the biological treatment of wastewater. A significant drawback of this process is the production of excess 'sludge', the disposal of which can account for 50–60% of the running costs of a plant. We investigate how the volume and mass of excess sludge produced is reduced by coupling the bioreactor to a sludge disintegration unit.

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1 Introduction

The activated sludge process (ASP) is the most commonly used method for the aerobic treatment of sewage and industrial wastewater. It contains an aerated bioreactor, where pollutants are degraded by bacteria, and a settling unit, to recycle activated sludge from the effluent stream into the bioreactor.

A product of the ASP is a heterogeneous sludge which must be purged from the system. Traditional methods for its disposal include incineration, landfill disposal and dumping at sea. These methods are becoming increasingly regulated and increasingly expensive. Consequently there is a growing interest in methods that reduce the amount of sludge produced in situ.

The simplest model for the ASP has two variables representing the concen-

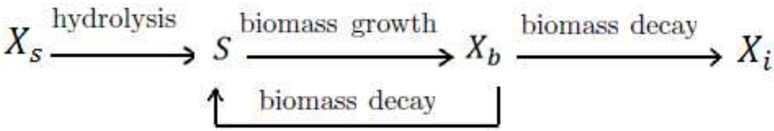

Figure 1: Microbial process in the bioreactor where S is the soluble substrate, X_b is the biomass, X_i is the non-biodegradable particulate matter, and X_s is the biodegradable particulate substrate.

trations of substrate and biomass [3]. Although this model is widely used to model the effluent concentration from the ASP it does not model sludge formation. An extension of this basic model adds an insoluble substrate component and allows a fraction of dead biomass to be recycled into the pool of insoluble substrate. This is the simplest model for sludge production and was proposed by Chung and Neethling [2].

In earlier work we investigated how the operation of the settling unit influences the amount of sludge formed in the bioreactor [1]. We extend this analysis by investigating how a sludge disintegration unit (SDU) is used to control the amount of sludge within the bioreactor.

2 Biochemistry

2.1 Microbial reactions

Figure 1 provides a schematic of the three biochemical processes in the model. The first biochemical process is the hydrolysis of biodegradable particulate substrate X_s to produce soluble substrate S:

$$X_s \xrightarrow{k_h} \alpha_h S \,,$$

where α_h is the yield factor for hydrolysis of insoluble organic compounds. The second biochemical process is the use of soluble organic materials S as a

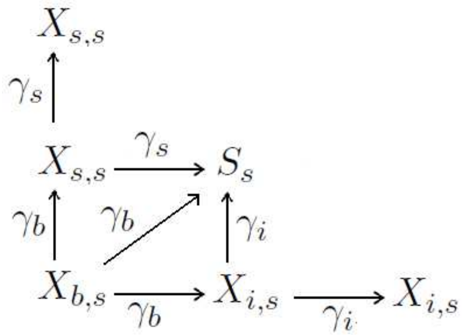

Figure 2: Disintegration processes occurring within the SDU. Nomenclature is provided in the text.

substrate for energy and growth by the biomass X_b :

$$S \xrightarrow{\mu(S)} \alpha_g X_b \,,$$

where α_g is the yield factor for growth of biomass. The final biochemical process is the death of biomass which adds to the pool of soluble substrate S and produces an inert residue X_i :

$$x_b \xrightarrow{k_d} f_i X_i + f_s \alpha_s S \,,$$

where f_i is the fraction of dead biomass converted to inert material, f_s is the fraction of dead biomass converted to soluble substrate, and α_s is the yield factor for conversion of dead biomass to soluble substrate. We assume that $f_i + f_s = 1$. Mass conservation imposes the restriction $0 < f_s \alpha_g \alpha_s \leqslant 1$.

Figure 2 shows the three disintegration reactions that occur within the SDU. The subscript s denotes a concentration inside the SDU. The first disintegration process is the conversion of biomass $X_{b,s}$ into non-biodegradable particulate $X_{i,s}$, biodegradable particulate substrate $X_{s,s}$ and soluble substrate S_s :

$$X_{b,s} \xrightarrow{\gamma_b} f_{p,s} X_{i,s} + (1 - \alpha - f_{p,s}) X_{s,s} + \alpha \alpha_s S_s \,, \tag{1}$$

where α is the solubilization efficiency of the SDU, α_s is the yield factor for conversion of dead biomass to soluble substrate, $f_{p,s}$ is the fraction of dead biomass converted to inert material in the SDU, γ_b is the disintegration rate of biomass, and $0 \leqslant \alpha \leqslant 1 - f_{p,s}$. The second disintegration process is the conversion of biodegradable particulate substrate $X_{s,s}$ into soluble substrate S_s and biodegradable particulate substrate $X_{s,s}$:

$$X_{s,s} \xrightarrow{\gamma_s} \alpha\alpha_h S_s + (1-\alpha)X_{s,s}\,, \tag{2}$$

where α_h is the yield factor for hydrolysis of insoluble substrate, and γ_s is the disintegration rate of biodegradable particulate substrate. The third disintegration process is the conversion of non-biodegradable particulate substrate $X_{i,s}$ into soluble substrate S_s and non-biodegradable particulate substrate $X_{i,s}$:

$$X_{i,s} \xrightarrow{\gamma_i} \beta_i\alpha_{\text{SDU}} S_s + (1-\beta_i)X_{i,s}\,, \tag{3}$$

where β_i is the conversion efficiency from non-biodegradable particulates to readily biodegradable (soluble) substrate, α_{SDU} is a yield factor for the conversion of inert material to soluble substrate through disintegration, and γ_i is the disintegration rate of non-biodegradable particulate substrate.

2.2 The dimensional model

We first provide all model equations in the bioreactor. All parameters are defined in Appendix A. The rate of change in the concentration of soluble substrate is

$$\frac{dS}{dt} = \frac{F}{V}(S_0 - S) + \frac{F}{V}D(S_s - S) + \alpha_h k_h X_s + f_s\alpha_s k_d X_b - \frac{X_b\mu(S)}{\alpha_g}\,. \tag{4}$$

The rate of change in the concentration of biomass is

$$\frac{dX_b}{dt} = -\frac{F}{V}X_b + \frac{F}{V}R(C-1)X_b + \frac{F}{V}D(X_{b,s} - X_b) + X_b\mu(S) - k_d X_b\,. \tag{5}$$

The rate of change in the concentration of non-biodegradable particulate material is

$$\frac{dX_i}{dt} = \frac{F}{V}(X_{i,0} - X_i) + \frac{F}{V}R(C-1)X_i + \frac{F}{V}D(X_{i,s} - X_i) + f_i k_d X_b \,. \tag{6}$$

The rate of change in the concentration of biodegradable particulate substrate is

$$\frac{dX_s}{dt} = \frac{F}{V}(X_{s,0} - X_s) + \frac{F}{V}R(C-1)X_s + \frac{F}{V}D(X_{s,s} - X_s) - k_h X_s \,. \tag{7}$$

The specific growth rate is

$$\mu(S) = \frac{\mu_m S}{K_s + S} \,. \tag{8}$$

The residence time is

$$\tau = \frac{V}{F} \,. \tag{9}$$

The chemical demand oxygen characterises the organic carbon content of wastewaters:

$$\text{COD} = S + \alpha_h X_s \,. \tag{10}$$

The total volatile suspended solids represents the solid organic carbon in the sludge:

$$\text{VSS} = X_t = X_b + X_i + X_s \,. \tag{11}$$

In practice, a target value for the VSS is set. We set the target value to $\text{VSS}_t = 12000\,\text{mg}\,\text{L}^{-1}$ [4].

We now provide all model equations in the sludge disintegration unit. Again, all parameters are defined in Appendix A. The rate of change in the concentration of soluble substrate is

$$\frac{dS_s}{dt} = \frac{F}{V_s}D(S - S_s) + \alpha\alpha_s\gamma_b X_{b,s} + \alpha\alpha_h\gamma_s X_{s,s} + \beta_i\alpha_{\text{SDU}}\gamma_i X_{i,s} \,. \tag{12}$$

The rate of change in the concentration of biomass is

$$\frac{dX_{b,s}}{dt} = \frac{F}{V_s}D(X_b - X_{b,s}) - \gamma_b X_{b,s} \,. \tag{13}$$

The rate of change in the concentration of non-biodegradable particulate material is

$$\frac{dX_{i,s}}{dt} = \frac{F}{V_s} D(X_i - X_{i,s}) + f_{p,s}\gamma_b X_{b,s} - \beta_i \gamma_i X_{i,s} \,. \tag{14}$$

The rate of change in the concentration of biodegradable particulate substrate is

$$\frac{dX_{s,s}}{dt} = \frac{F}{V_s} D(X_s - X_{s,s}) + (1 - \alpha - f_{p,s})\gamma_b X_{b,s} - \alpha\gamma_s X_{s,s} \,. \tag{15}$$

The most important parameter associated with the SDU is the sludge disintegration factor D.

2.3 The dimensionless model

By introducing dimensionless variables for time and concentrations of the substrate, microorganism and particulates, the model equations are written in dimensionless form. An asterisk superscript indicates a dimensionless variable.

The dimensionless equations which model the reactor are

$$\frac{dS^*}{dt^*} = \frac{1}{\tau^*}(S_0^* - S^*) + \frac{D}{\tau^*}(S_s^* - S^*) + \alpha_{gh} k_h^* X_s^* + f_s \alpha_{gs} k_d^* X_b^* - \frac{S^* X_b^*}{1 + S^*} \,, \tag{16}$$

$$\frac{dX_b^*}{dt^*} = \frac{-1}{\tau^*} X_b^* + \frac{R^*}{\tau^*} X_b^* + \frac{D}{\tau^*}(X_{b,s}^* - X_b^*) + \frac{X_b^* S^*}{1 + S^*} - k_d^* X_b^* \,, \tag{17}$$

$$\frac{dX_i^*}{dt^*} = \frac{1}{\tau^*}(X_{i,0}^* - X_i^*) + \frac{R^*}{\tau^*} X_i^* + \frac{D}{\tau^*}(X_{i,s}^* - X_i^*) + f_i k_d^* X_b^* \,, \tag{18}$$

$$\frac{dX_s^*}{dt^*} = \frac{1}{\tau^*}(X_{s,0}^* - X_s^*) + \frac{R^*}{\tau^*} X_s^* + \frac{D}{\tau^*}(X_{s,s}^* - X_s^*) - k_h^* X_s^* \,. \tag{19}$$

In addition,

$$\text{COD}^* = S^* + \alpha_{g,h} X_s^* \quad \text{and} \quad \text{VSS}^* = X_b^* + X_i^* + X_s^* \,. \tag{20}$$

The equations which model the sludge disintegration unit are

$$\frac{dS_s^*}{dt^*} = \frac{DV^*}{\tau^*}(S^* - S_s^*) + \alpha\alpha_{gs}\gamma_b^* X_{b,s}^* + \alpha\alpha_{gh}\gamma_s^* X_{s,s}^* + \beta_i\alpha\alpha_{g\text{SDU}}\gamma_i^* X_{i,s}^*\,, \quad (21)$$

$$\frac{dX_{b,s}^*}{dt^*} = \frac{DV^*}{\tau^*}(X_b^* - X_{b,s}^*) - \gamma_b^* X_{b,s}^*\,, \quad (22)$$

$$\frac{dX_{i,s}^*}{dt^*} = \frac{DV^*}{\tau^*}(X_i^* - X_{i,s}^*) + f_{p,s}\gamma_b^* X_{b,s}^* - \beta_i\gamma_i^* X_{i,s}^*\,, \quad (23)$$

$$\frac{dX_{s,s}^*}{dt^*} = \frac{DV^*}{\tau^*}(X_s^* - X_{s,s}^*) + (1 - \alpha - f_{p,s})\gamma_b^* X_{b,s}^* - \alpha\gamma_s^* X_{s,s}^*\,. \quad (24)$$

The target value for VSS^* is

$$\text{VSS}_t^* = \frac{\text{VSS}_t}{\alpha_g k_s} = \frac{12000}{\alpha_g k_s} = 77.071\,. \quad (25)$$

3 Results

Alharbi et al. [1] found the steady-state solutions of the ASP without a SDU, equations (16)–(19) with $D = 0$, and their stability was determined as a function of the residence time τ^*. These solutions were used to investigate the steady-sludge VSS concentration as a function of the operation of the settling unit through the effective recycle parameter R^*, constrained by $0 \leqslant R^* \leqslant 1$.

We assume that the disintegration processes in the SDU are much quicker than the biochemical processes within the bioreactor so that the SDU effectively operates at steady-state. This reduces the ASP-SDU model to four differential equations for the ASP and four algebraic equations for the SDU. As the steady-state solution within the SDU is readily found, the ASP-SDU model reduces to four non-linear differential equations.

3.1 Steady-state solutions and their stability

The steady-state solution within the SDU is comprised of two branches: a washout solution branch representing process failure where $X_b^* = 0$; and a no-washout branch where $X_b^* \neq 0$. The lengthy expressions for these solution branches are omitted. The stability of both solution branches must be determined numerically.

3.2 Asymptotic solutions

At large residence times along the no-washout (NW) branch

$$S^*_{\text{NW}} \approx \frac{k_d^*}{1-k_d^*} + \frac{1-R^*+D}{(k_d^*-1)^2\tau^*} + \mathcal{O}\left(\frac{1}{\tau^{*2}}\right), \tag{26}$$

$$X^*_{b,\text{NW}} \approx \frac{a_0}{\tau^*} + \mathcal{O}\left(\frac{1}{\tau^{*2}}\right), \tag{27}$$

$$X_i^* \approx \frac{X^*_{i,0}}{1-R^*+D} + \frac{f_i k_d^* a_0}{1-R^*+D} + \frac{a_1}{\tau^*} + \mathcal{O}\left(\frac{1}{\tau^{*2}}\right), \tag{28}$$

$$X_s^* \approx \frac{X^*_{s,0}}{k_h^*\tau^*} + \mathcal{O}\left(\frac{1}{\tau^{*2}}\right), \tag{29}$$

$$\text{COD}^*_{\text{NW}} \approx \frac{k_d^*}{1-k_d^*} + \left(\frac{1-R^*+D}{(k_d^*-1)^2} + \frac{\alpha_{gh}X^*_{s,0}}{k_h^*}\right)\frac{1}{\tau^*} + \mathcal{O}\left(\frac{1}{\tau^{*2}}\right), \tag{30}$$

$$\text{VSS}^*_{\text{NW}} \approx \frac{X^*_{i,0} + f_i k_d^* a_0}{1-R^*+D} + \left(a_0 + a_1 + \frac{X^*_{s,0}}{k_h^*}\right)\frac{1}{\tau^*} + \mathcal{O}\left(\frac{1}{\tau^{*2}}\right). \tag{31}$$

where

$$a_0 = \frac{(1-k_d^*)b_0 - k_d^*(1-R^*+D)}{k_d^*(1-k_d^*)\left[(1-R^*+D)(1-f_s\alpha_{gs}) - D\alpha\alpha_{gs\text{DU}}f_i\right]}, \tag{32}$$

$$b_0 = D\alpha\alpha_{gs\text{DU}}X^*_{i,0} + (1-R^*+D)\left(\alpha_{gh}X^*_{s,0} + S_0^*\right). \tag{33}$$

The expression for a_1 is not included due to its length. It is independent of the disintegration rate γ_i.

4 Discussion

We consider the case when the disintegration rates in the biomass, biodegradable particulate and non-biodegradable particulate are equal: $\gamma_j = \gamma$ for $j = i, b, s$; and investigate the effect of the SDU on sludge formation.

4.1 Sludge formation in the absence of a SDU ($D = 0$)

The accumulation of sludge in the ASP in the absence of a SDU was investigated by Alharbi et al. [1]. There are five generic steady-state response diagrams for the sludge content as a function of the residence time τ^* as the effective recycle parameter R^* is varied. For the first four cases there is a critical value of the residence time τ^*_{sludge} such that if $\tau^* > \tau^*_{\text{sludge}}$, then the steady-state value of the volatile suspended solids is always below the target value. These cases are highly desirable, but are rarely achieved in practice.

For high values of the effective recycle parameter, $0.7387 \leqslant R^* \leqslant 1$, the response diagram is shown in Figure 3 (case five). The steady-state sludge content is *always* higher than the target value VSS^*_t. This is undesirable.

4.2 Sludge formation in the presence of a SDU ($D > 0$)

This section concentrates on how the operation of the SDU effects the steady-state diagram for 'case five'. In this case the sludge content is above the target value for *any* value of the residence time.

Figures 4 and 5 shows the steady-state sludge content over time, for different decay rates γ and for two values of the effective recycle ratio R^*. In Figure 4 where $R^* = 0.6$, as the disintegration rate γ increases the steady-state diagram transitions into case four. In Figure 5 where $R^* = 0.9$, the SDU is less successful than when $R^* = 0.6$. Although the amount of sludge within

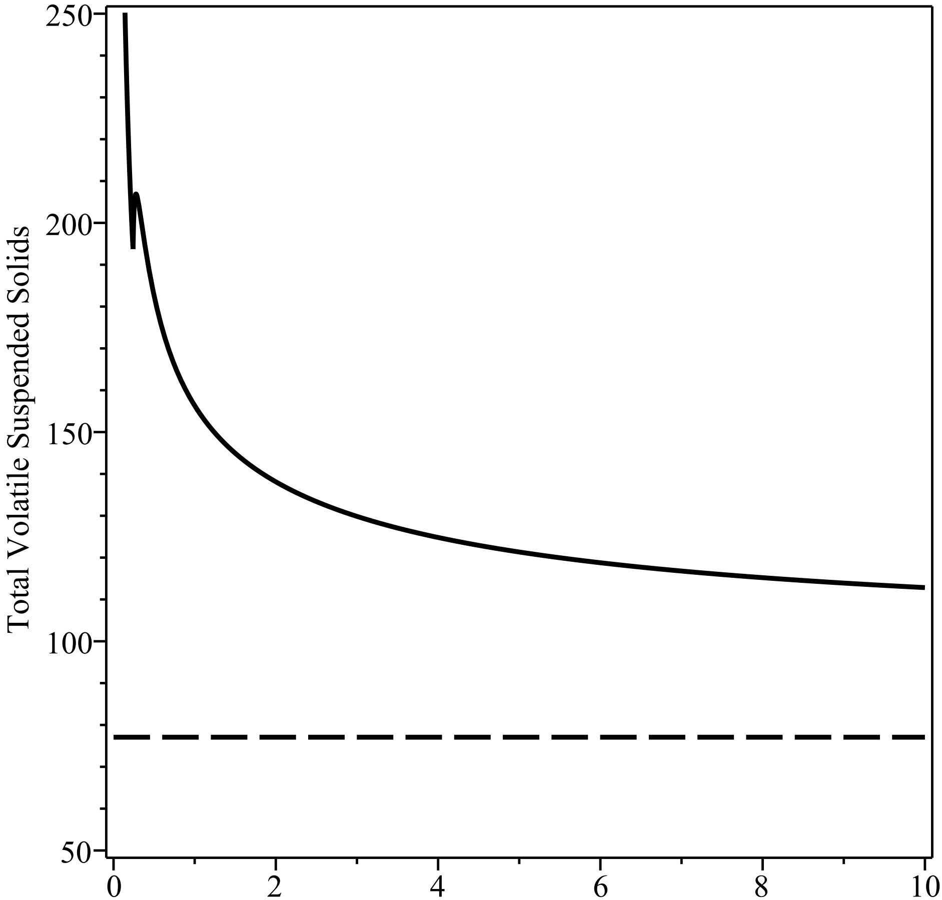

Figure 3: Steady state diagram for the volatile suspended solids (case 5) in the absence of a SDU with $R^* = 0.8$. The horizontal line denotes the target value of the volatile suspended solids VSS_t^*.

the bioreactor is reduced as the disintegration rate increases, the response diagram remains 'case five'.

When there is no SDU the transition from case four to case five occurs when the effective recycle ratio is $R^*_{45} = 0.7387$. Our numerical investigation revealed the operation of a SDU increases this transition value. We found the transition value to be independent of the scaled volume of the SDU V^* and the disintegration rate γ. For the parameter values used in Figures 4 and 5 ($D = 0.1$) the transition value is $R^*_{45} = 0.8385$. Thus the SDU allows higher values of the effective recycle parameter without the production of excess sludge. In Figure 5 the effective recycle parameter $R^* = 0.9$ is higher than the new critical value $R^*_{45} = 0.8385$.

The transition value R^*_{45} is determined from the VSS value in the limit of infinite residence time. Equation (31) show that this value is independent of the sludge disintegration rates, γ_b, γ_i and γ_s, and the conversion efficiency β_i; it depends upon D, α and α_{gsDU}. Only the sludge disintegration factor D can be changed. From equation (31) the VSS at infinity is equal to the target value when

$$Q(D) = a_D D^2 + b_D D + c_D = 0\,, \tag{34}$$

where the coefficients are

$$\begin{aligned}
a_D &= -\mathrm{VSS}^*_t(1-k^*_d)(1-\alpha\alpha_{gsDU}f_i - f_s\alpha_{gs}) \leqslant 0\,,\\
b_D &= (1-k^*_d)\{(1-f_s\alpha_{gs})X^*_{i,0} + (\alpha_{gh}X^*_{s,0} + S^*_0)f_i\\
&\quad - \mathrm{VSS}^*_t\,(1-R^*)\,[2(1-f_s\alpha_{gs}) - \alpha\alpha_{gsDU}f_i]\} - f_i k^*_d\,,\\
c_D &= (1-R^*)\big((1-k^*_d)\{(1-f_s\alpha_{gs})[X^*_{i,0} - \mathrm{VSS}^*_t(1-R^*)]\\
&\quad + (\alpha_{gh}X^*_{s,0} + S^*_0)f_i\} - f_i k^*_d\big).
\end{aligned}$$

When $D = 0$ [1] we have $R^*_1 = 1$ and

$$R^*_2 = \frac{(1-k^*_d)[(1-f_s\alpha_{gs})(\mathrm{VSS}^*_t - X^*_{i,0}) - (\alpha_{gh}X^*_{s,0} + S^*_0)f_i] + f_i k^*_d}{(1-k^*_d)(1-f_s\alpha_{gs})\mathrm{VSS}^*_t}\,.$$

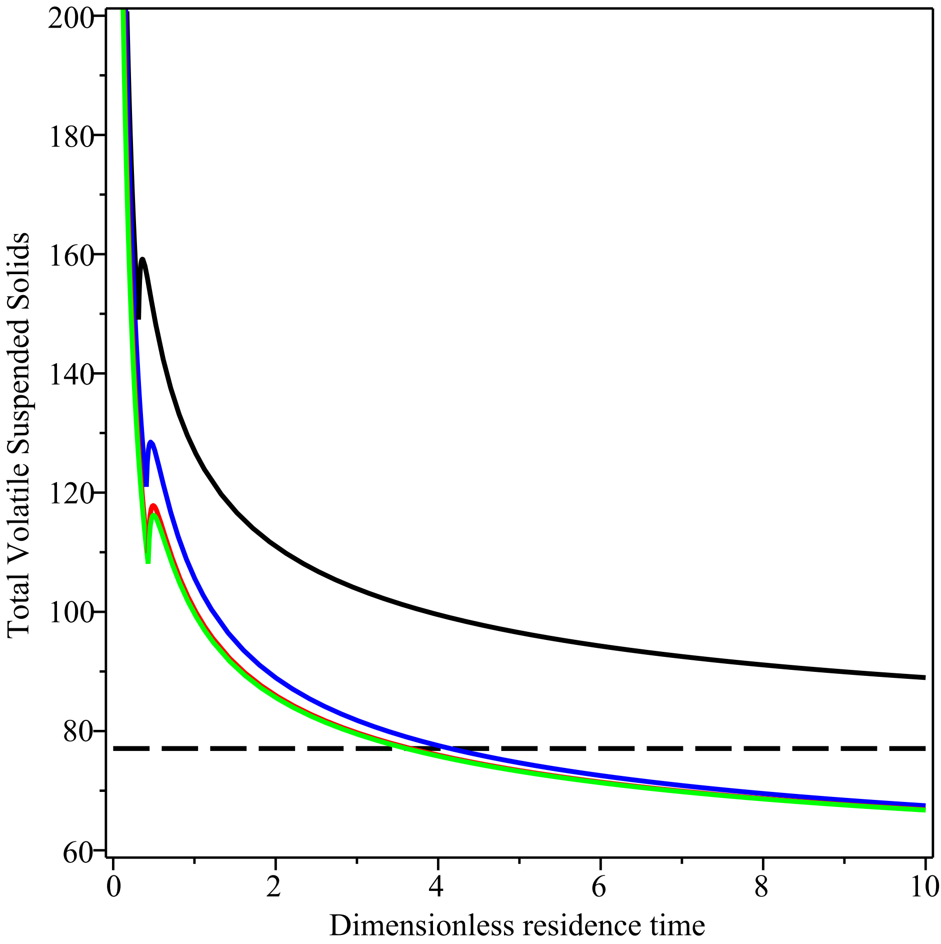

Figure 4: The volatile suspended solids concentration as a function of the residence time. Parameter values are $D = 0.1$, $V^* = 1$, $R^* = 0.6$ and $\gamma = 0, 0.1, 1, 10$ for black, blue, red and green, respectively.

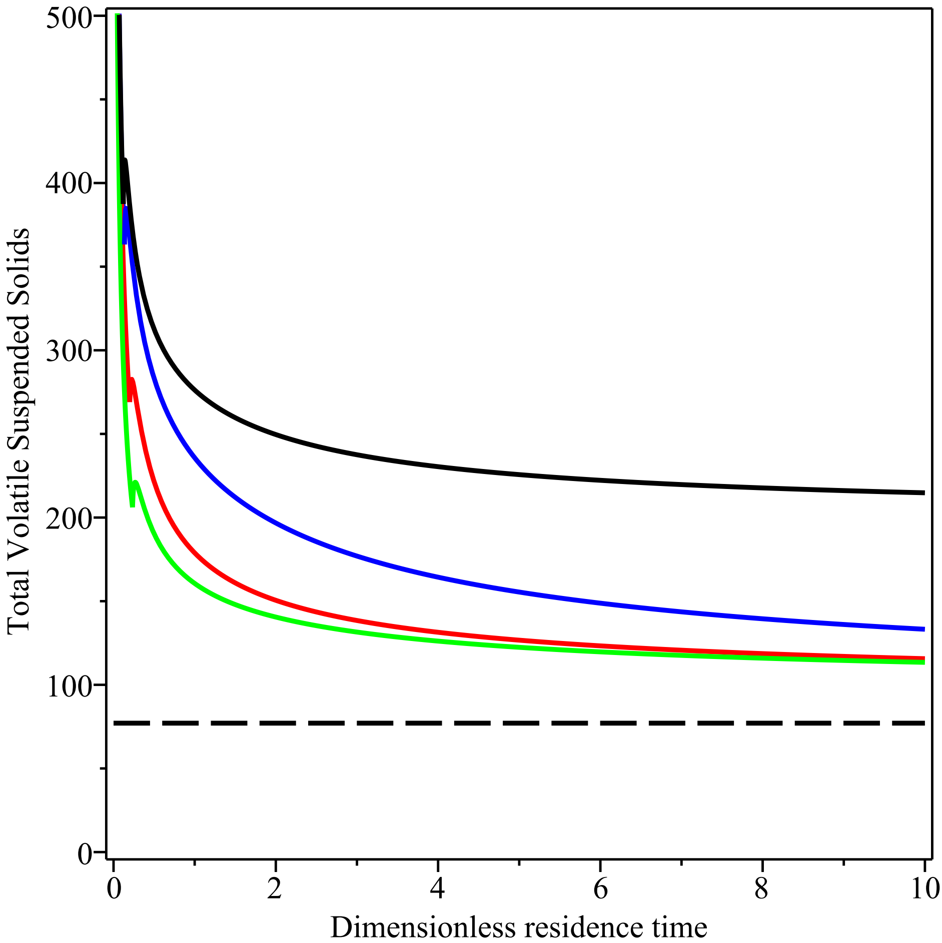

Figure 5: The volatile suspended solids concentration as a function of the residence time. Parameter values are $D = 0.1$, $V^* = 1$, $R^* = 0.9$ and $\gamma = 0, 0.1, 0.35, 10$ for black, blue, red and green, respectively. For $1 \leqslant \gamma \leqslant 10$ plots are visually indistinguishable.

When $R^* = 1$ we have, from equation (34), $D_1 = 0$ and

$$D_2 = \frac{(1-k_d^*)[(1-f_s\alpha_{gs})X_{i,0}^* + (\alpha_{gh}X_{s,0}^* + S_0^*)f_i] - f_i k_d^*}{\text{VSS}_t^*(1-k_d^*)(1-\alpha\alpha_{\text{gsDU}}f_i - f_s\alpha_{gs})}\,.$$

Figure 6 plots equation (34). To the left of the line is the desirable region where the VSS at infinite residence time is smaller than the target value. To the right of the line the VSS at infinite residence time is larger than the target value. In the former region, as the disintegration rate γ is increased from zero, transitions in the steady-state diagram occur, similar to Figure 4. In the latter region, the amount of sludge formed decreases as the disintegration rate increases. However, the steady-state diagram is always 'case five', similar to Figure 5.

It is surprising that the curve defined by equation (34), a quadratic equation, is a straight line. However, examination of the discriminant of the quadratic equation (34) reveals that it is essentially independent of the effective recycle parameter over the range $0 \leqslant R^* \leqslant 1$. It then follows that there is a linear relationship between the sludge disintegration factor D and the effective recycle parameter R^*.

5 Conclusions

We extended a model for sludge formation in the activated sludge process [1] to include a sludge disintegration unit. As the disintegration rate γ increases the amount of sludge in the bioreactor decreases. Two forms of behaviour are found as the disintegration rate is increased, see Figures 4 and 5. The transition between the two behaviours is governed by the VSS value when the residence time is infinite. This is independent of the disintegration rate γ. For a fixed value of the effective recycle parameter R^* there is a critical value of the sludge disintegration factor D. If D is sufficiently high, then the sludge content is below the target value for sufficiently large values of the residence

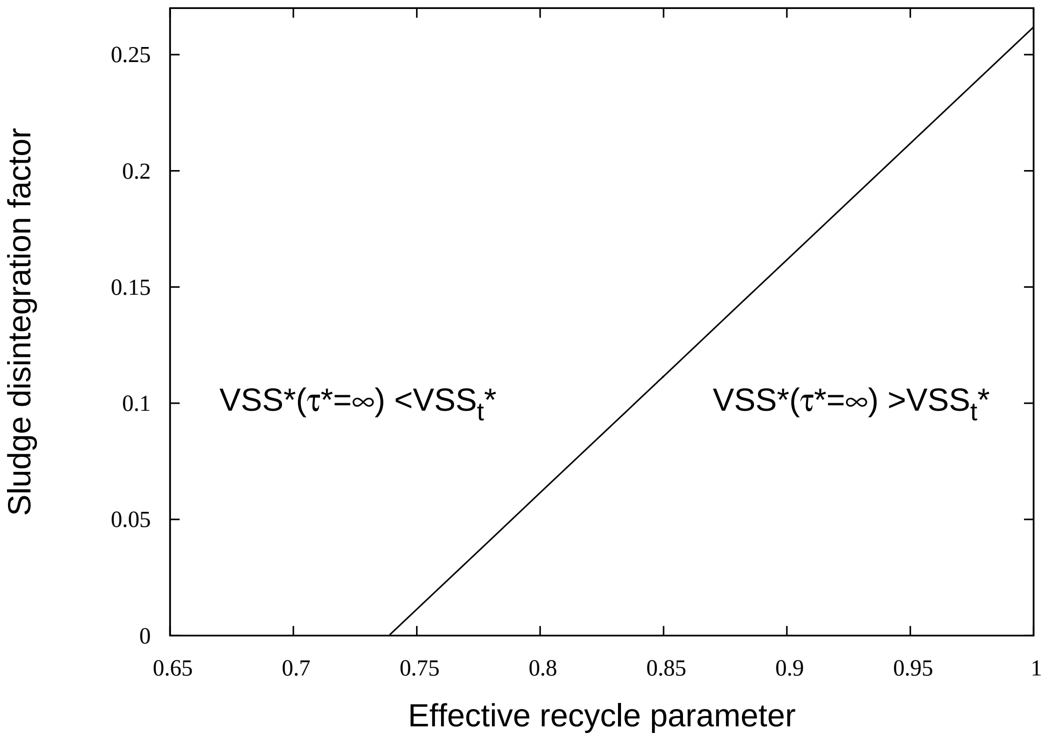

Figure 6: The transition between the behaviour shown in Figures 4 and 5 as a function of the effective recycle parameter R^* and the sludge disintegration factor D when $V^* = 1$. This transition is governed by equation (34).

time. If D is below the critical value, then the the sludge content is always above the target value.

In future work we aim to comprehensively analyse the behaviour of the activated sludge process for finite values of the disintegration rate γ.

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A Nomenclature

Here we provide definitions and typical units for all parameters. We denote the units of soluble substrate by $|S|$ and the units of biomass, non-biodegradable particulate matter and biodegradable particulate substrate by $|X|$. Typical units are $|S| = (\text{mg COD})\text{L}^{-1}$ and $|X| = (\text{mg VSS})\text{L}^{-1}$.

C	Recycle concentration factor	(–)
COD	Chemical oxygen demand	($|S|$)
D	Sludge disintegration factor	(–)

F	Bioreactor flow rate	$(\mathrm{dm}^3\,\mathrm{hr}^{-1})$
K_s	Monod constant	$(\lvert S \rvert)$
R	Recycle ratio	$(-)$
S	Substrate concentration in bioreactor	$(\lvert S \rvert)$
S_s	Substrate concentration in SDU	$(\lvert S \rvert)$
S_0	Substrate concentration in feed	$(\lvert S \rvert)$
$S_k^* = S_k/S_0$	Dimensionless substrate concentration	$(-)$
V	Bioreactor volume	(dm^3)
V_s	SDU volume	(dm^3)
$V^* = V/V_s$	Scaled SDU reactor volume	$(-)$
X_b	Biomass concentration in bioreactor	$(\lvert X \rvert)$
$X_{b,s}$	Biomass concentration in SDU	$(\lvert X \rvert)$
X_i	Concentration of non-biodegradable particulate materials in bioreactor	$(\lvert X \rvert)$
$X_{i,s}$	Concentration of non-biodegradable particulate materials in SDU	$(\lvert X \rvert)$
X_s	Concentration of biodegradable particulate substrate in bioreactor	$(\lvert X \rvert)$
$X_{s,s}$	Concentration of biodegradable particulate substrate in SDU	$(\lvert X \rvert)$
$X_{j,0}$	Feed concentration $(j = b, i, s)$	$(\lvert X \rvert)$
X_t	Total biomass concentration	$(\lvert X \rvert)$
$X_k^* = X_k/(\alpha_g K_s)$	Dimensionless concentration	$(-)$
k_d	Death coefficient	(hr^{-1})
$k_d^* = k_d/\mu_m$	Dimensionless death coefficient	$(-)$
k_h	Hydrolysis rate of insoluble organic compounds	(hr^{-1})
$k_h^* = k_h/\mu_m$	Dimensionless hydrolysis rate	$(-)$
f_s	Fraction of dead biomass converted to soluble substrate	$(-)$
f_i	Fraction of dead biomass converted to inert material	$(-)$
t	Time	(hr^{-1})

$t^* = \mu_m t$	Dimensionless time	$(-)$
α_g	Yield factor for growth of biomass	$(\lvert X\rvert\lvert S\rvert^{-1})$
$\alpha_{g\text{SDU}} = \alpha_g\alpha_{\text{SDU}}$	Dimensionless yield coefficient in SDU	$(-)$
$\alpha_{gh} = \alpha_h\alpha_g$	Dimensionless yield coefficient in bioreactor	$(-)$
$\alpha_{gs} = \alpha_g\alpha_s$	Dimensionless yield coefficient in bioreactor	$(-)$
α_h	Yield factor for hydrolysis of insoluble organic compounds	$(\lvert S\rvert\lvert X\rvert^{-1})$
α_s	Yield factor: conversion of dead biomass to soluble substrate	$(\lvert S\rvert\lvert X\rvert^{-1})$
γ_b	Disintegration rate of biomass in SDU	(hr^{-1})
γ_i	Disintegration rate of non-biodegradable particulate substrate in SDU	(hr^{-1})
γ_s	Disintegration rate of biodegradable particulate substrate in SDU	(hr^{-1})
$\gamma_k^* = \gamma_k/\mu_m$	Dimensionless disintegration rate $(k = b, i, s)$	$(-)$
$\mu(S)$	Specific growth rate model	(hr^{-1})
μ_m	Maximum specific growth rate	(hr^{-1})
τ	Residence time	(hr)
$\tau^* = \mu_m\tau$	Dimensionless residence time	$(-)$

The following gives some typical parameter values [2].

$K_s = 5,190\,(\text{mg COD})\text{L}^{-1}$	$S_0 = 10,360\,(\text{mg COD})\text{L}^{-1}$
$X_{i,0} = 2,810\,(\text{mg VSS})\text{L}^{-1}$	$X_{s,0} = 22,59\,(\text{mg VSS})\text{L}^{-1}$
$f_s = 0.8$	$k_d = 0.015\,\text{day}^{-1}$
$k_h = 1.1\,\text{day}^{-1}$	$\alpha_g = 0.03\,(\text{mg VSS})/(\text{mg COD})$
$\alpha_h = 1.88\,(\text{mg COD})/(\text{mg VSS})$	$\alpha_S = 1.42\,(\text{mg COD})/(\text{mg VSS})$
$\mu_m = 0.22\,\text{day}^{-1}$	

The following gives some typical dimensionless parameters.

$S_0^* = 1.996$	$X_{s,0}^* = 145.0857$	$X_{i,0}^* = 18.0475$	$k_d^* = 0.068$
$k_h^* = 5$	$\alpha_{g,h} = 0.0564$	$\alpha_{g,s} = 0.0426$	$\text{COD}_0^* = 10.1788$

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