ANZIAM J. 56 (MISG2014) pp.M1–M29, 2016

Seaweed cultivation and the remediation of by-products from ethanol production: a glorious green growth

Winston L. Sweatman[1] Geoff Mercer[2] John Boland[3]
Nicole Cusimano[4] Ava Greenwood[5] Kristen Harley[6]
Peter van Heijster[7] Peter Kim[8] Joe Maisano[9]
Mark Nelson[10] Graeme Pettet[11]

(Received 18 March 2015; revised 29 April 2016)

Abstract

Mathematical models are presented for the cultivation of seaweed. These relate to a mathematics-in-industry project to grow seaweed crops to consume by-products from commercial ethanol production. An initial model illustrates the process. Then, the potential is demonstrated with a more detailed feasibility study and a simple financial model. The growth of seaweed with time is described using various models utilising differential equations. These include factors such as solar radiation and the nitrogen content of the seaweed.

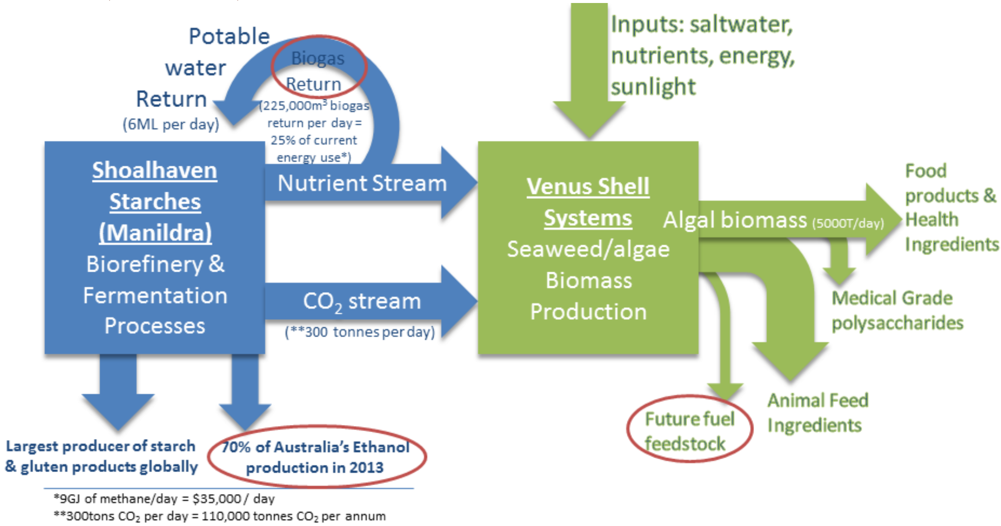
http://journal.austms.org.au/ojs/index.php/ANZIAMJ/article/view/9402 gives this article, Published May 26, 2016, as part of the Proceedings of the 2014 Mathematics and Statistics in Industry Study Group. ISSN 1445-8810.

Contents

1 Introduction

Venus Shell Systems produces high quality seaweed that is suitable for various food and medical purposes. The seaweed is grown in land-based tanks. During its growth the seaweed needs to be supplied with various nutrients and sunlight. The 2014 MISG project considered how these nutritional requirements might be met by growing the seaweed on site at the Manildra Biorefinery, New South Wales, using waste byproducts (retentate) from large-scale ethanol distillation. When required, additional nutrients are provided from other sources. Seaweed production would meanwhile allow safe and efficient disposal of the retentate removing this constraint to expansion of the ethanol distillery. Figure 1 has been produced by the industry representative and illustrates a

Figure 1: Flow diagram for production at Venus Shell Systems and Shoalhaven Starches (Manildra). Figure courtesy Venus Shell Systems.



combined production flow from Venus Shell Systems and Shoalhaven Starches (Manildra).

Currently, retentate is disposed of by pasture irrigation. The disposal is constrained by the land area available to the ethanol distillery. Essentially the nutrients are converted into beef. The nutrient conversion rate of seaweed growth is significantly higher than that of grazing cattle (by a factor of about 20 times) and so needs less area to dispose of the same amount of retentate. However, there are higher set up and maintenance costs to consider for the seaweed production.

Key inputs for seaweed production, which are also significant in the biorefinery output, are nitrogen-containing products (nitrate and ammonium compounds), carbon dioxide and trace elements. Growing conditions in the tanks are maintained relatively consistent on a timescale of weeks or even months, although some variation occurs with weather conditions and day and night.

Carbon dioxide, produced by ethanol production, is bubbled into the tanks. Calculations indicate that the supply of carbon dioxide may be considered limitless for the present levels of production. The carbon dioxide concentration within the tanks is monitored using pH levels and kept at an optimal value. The introduction of carbon dioxide bubbles also has the effect of continuously mixing the contents of the seaweed-growing tanks removing variation with depth. Regular harvest of seaweed also maintains the optimal growing density: there should be sufficient seaweed to utilise the inputs without producing too much shading of the lower tank. Temperature and light vary significantly between summer and winter impacting on seaweed growth rates.

A variety of aspects of the proposal have been considered in separate investigations within this paper. A direct matching has been made between the current or potential retentate supply and seaweed production. From this a financial model has been built. The differences between growing rates in summer and winter are considered as are the potential effect of cloudy days. Differential equations are used for various models to consider variation in the seaweed production with time.

2 A simple first model

We begin with a relatively simple model for initial illustration. This relates the consumption of nitrogen coming from the retentate stream with the production of seaweed. Key assumptions based on information provided by Venus Shell Systems are:

- The retentate stream of nitrogen supplied $11,000$ kg per day or $77,000$ kg per week.
- The growth of seaweed biomass is quantified on a weekly basis and depends on seasonal effects. In particular, 1 kg of wet seaweed biomass grows weekly to 2 kg in winter and to 3 kg in summer.

- The nitrogen content is a constant proportion of the seaweed dry weight, approximately 5%. Because 80% of wet seaweed is water, the nitrogen content is 1% of the seaweed wet weight.

We assume that the seaweed is grown in ideal conditions and the supply of nutrients including carbon is kept constant over time. Moreover, tanks are continually mixed so that the entire biomass is exposed to the same amount of light and we may neglect the effects of depth within the tanks. Finally, the biomass is automatically harvested so that the quantity of seaweed in the tanks is kept at $1\,\mathrm{kg\,m^{-3}}$, the value leading to maximum productivity.

The time independence of most of the processes enables us to develop a simple relationship in terms of the weekly production rate. Let N be the nitrogen consumed (or required) per week and S be the total wet seaweed biomass retained in the tanks. Based on the information provided by Venus Shell Systems, we define the weekly production rate Q (which depends on the season) so that 1 kg of wet seaweed produces $(1 + \mathsf{Q})$ kg of wet seaweed per week. Therefore the total seaweed production per week is $\mathsf{S} \times \mathsf{Q}$. This quantity also corresponds to the amount of seaweed harvested per week to maintain optimal productivity. Recalling that the nitrogen content in wet seaweed is 1% of the biomass weight, the total nitrogen consumption per week is 1% of $\mathsf{S} \times \mathsf{Q}$. Hence,

$$\mathsf{S} \times \mathsf{Q} = 100\mathsf{N}. \tag{1}$$

For a particular season, that is given a particular value of Q, equation (1) is used to obtain the total amount of seaweed that has to be retained by the tanks in order to consume the constant supply of nitrogen ($\mathsf{N}_0 = 77,000\,\mathrm{kg}$ per week). For example, in winter (when $\mathsf{Q} = 1$)

$$\mathsf{S} = \frac{100\ \mathsf{N}_0}{\mathsf{Q}} = 77 \times 10^5\,\mathrm{kg}. \tag{2}$$

Knowing the size of each tank, we estimate the number of tanks required to hold the seaweed and keep production at its optimum. In particular, 1 kg of wet seaweed occupies $1\,\mathrm{m}^3$ of water and each tank is 9 m long, 1.2 m wide and

Figure 2: Example of how the weekly production rate profile is affected by seasonality.

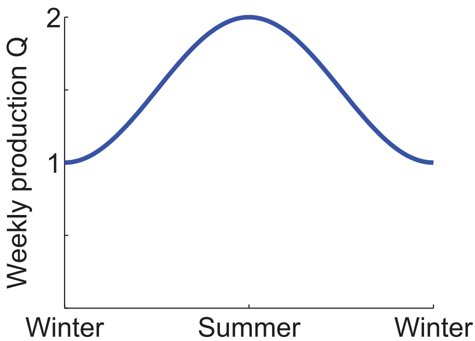
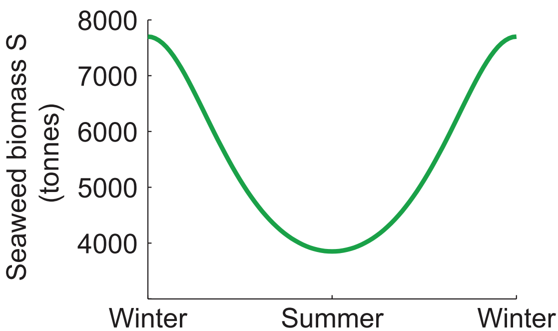


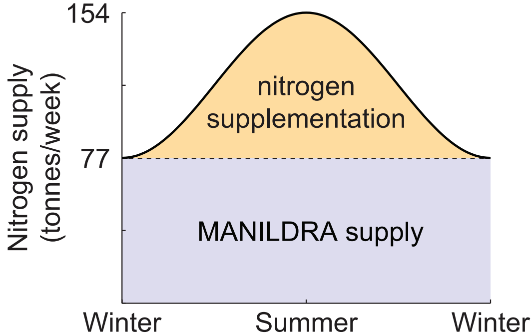
Figure 3: Adjustment of seaweed biomass in the tanks to maintain constant nitrogen consumption.



0.6 m deep, resulting in a capacity of $6.48\,\mathrm{m}^3$. Hence, the quantity of seaweed given in equation (2) requires approximately 1.19 million tanks. In summer, because of the higher production rate ($\mathrm{Q} = 2$), we only need 38.5×10^5 kg wet seaweed to process the same amount of nitrogen.

Figure 2 depicts an example of seasonally varying weekly production rate Q. Figure 3 displays the consequent inverse adjustment of seaweed biomass in the tanks in order to maintain the nitrogen consumption constant.

Figure 4: Example of augmented nitrogen supply to maximize seaweed production.



Depending on the strategy adopted by Venus Shell systems, halving the total amount of seaweed during summer might result in only half of the tanks being used or all the tanks being filled to half capacity. Alternatively, the company could decide to use all the tanks to full capacity and maximize seaweed production throughout the year by adding extra nitrogen to the existing constant retentate supply when the growth is more rapid. Figure 4 illustrates an example of augmented supply, in agreement with the production rate profile given in Figure 2.

3 Feasibility study and a simple financial model

The MISG team worked with two sets of retentate data. One was the current output at Manildra (Scenario A) and the other was a potential new retentate stream that would follow extended production at the refinery (Scenario B). These outputs were matched with the input demands of the seaweed production unit. The primary aim was to consume all of the nitrogen supplied. The secondary aim was to utilise as far as possible other trace elements present in

Table 1: Trace constituents of the Manildra retentate for existing and extended production.

Trace constituent	Existing production (Scenario A) kg/day	Extended production (Scenario B) kg/day
Na	549	4,202
K	119	3,121
Mg	1,040	12
Ca	44	97
Cl	330	2,230
N	22	11,075
P	5	1,742
CO_2	abundant	300,000
SO_4	227	31,291
H_2O	1,500,000	6,800,000

the retentate. Building off this model the financial aspects of the project were considered. The spreadsheet program EXCEL was used for this task. Table 1 shows constituent outputs (trace elements or compounds) of the existing and extended production at the refinery. The new extended production uses new techniques and hence there is a different balance of retentate constituents. Calculations were made of trace element take-up on a per hectare basis with determination of surplus and deficient trace elements in the retentate. There was some consideration of season, seaweed type and the utilisation of seawater as an additional source of water and nutrients.

The investigations concluded that in Scenario A the nitrogen in the retentate could be completely consumed with less than a tenth of the land mass currently required for pasture dispersal with beef production. All trace elements could be consumed using a fraction of the present land use. As observed in Section 2, the higher summer yield could be used for increased production. In Scenario B, the land used to consume all the nitrogen greatly exceeded that required under Scenario A, although this land is apparently available. Additional trace elements must be added for this case.

As well as needing the consumption of the Manildra waste products to be feasible, the project needs to be financially viable. It needs to be profitable for Venus Shell Systems to produce the seaweed, and also worthwhile for the Manildra biorefinery to work with Venus Shell Systems rather than dispose of the waste products on cattle pasture. To evaluate seaweed production from the Manildra waste products, an economic model was assembled that assigns cost to each action to be taken in the process, whether that be adding nutrients where deficient or 'scrubbing' unwanted elements from the waste water. The model is most detailed in the areas of actual seaweed production. However, it includes high level estimates of operational costs and benefits of the exercise to the Manildra biorefinery. One of the key drivers of feasibility is that we are expecting to obtain land and most nutrients at zero cost, therefore the primary cost to the business would be the installation of the tanks, pipes and control systems required to grow the seaweed.

Initial efforts were focused on a single species of seaweed and using only the Manildra retentate flow. However, there was some consideration of possible extensions to include different seaweed types and the addition of seawater.

Modelling using approximate costs, land use ratios, growth rates, et cetera, indicated that the venture is theoretically feasible. Nevertheless, there are scaling challenges moving from Scenario A to Scenario B. If the plant is built at the scale required to accept all of the waste under Scenario B (thus enabling the upgraded ethanol producing process), then the land use and capital requirements are enormous, and despite on-paper profitability, the assumptions of finance available and supply chain efficiency may break down. However, the industry representative noted at the meeting that the potential for consumption of high-grade seaweed products was also huge. For example, factories are being built to produce food for fish farming that require inputs at an even larger scale. If the seaweed production facility is built at a smaller scale, then the benefit is less clear.

3.1 Simple financial model

The structure of a simple financial model is now presented. If, taking into account all costs, the gross margin on each kilogram of seaweed produced is positive, then the venture is theoretically feasible. While the economic model itself features minimal complexity, it relies heavily on inputs from other models, including proportions from biological growth models, and cost of supplying or mitigating nutrients.

Once the take-up rates of elements by the seaweed and the chemical content of the retentate is known, a simple high-level financial model will be specified. It should be stressed that the model below is simplistic and relies heavily on certain assumptions. It considers most variables as a static average over a period of time and does not consider more complex factors such as flow and growth rates. The model contains a simple seasonality factor but does not consider long term climatic variations.

The gross margin, in Australian dollars, for every kilogram of seaweed produced is

$$\begin{aligned} M_q &= S - C - \frac{\sum_i \left([n^T_{iq} - n^F_i]^+ \times n^A_i\right) + \sum_i \left([n^F_i - n^T_{iq}]^+ \times n^R_i\right)}{P_q \times L_a} \\ &\quad - \frac{\frac{d_q}{365}\left([L_a \times L_c - B]^+ + T + O\right)}{P_q \times L_a} \end{aligned} \tag{3}$$

where:

M_q = Margin per kg of seaweed produced, for season q, in \$;

S = Average sale price of seaweed, per kg of dry weight;

C = Cost of drying, packing and transporting seaweed to market, in \$ per kg;

i = Nutrient present in retentate flow or required by seaweed, where $i \in \{$C, N, P, K, Na, Mg, Ca, S, Cu, Zn, Mn, Fe, B, Mb, Co, Si$\}$;

n^T_{iq} = Take-up of nutrient i by the seaweed for season q, in kg per season;

n^F_i = Mass of nutrient i in retentate flow, in kg per season;

$[a]^+$ = max(a, 0);

n^A_i = Cost to add deficient nutrient i, in \$ per kg;

n^R_i = Cost to mitigate (remove) accumulating nutrient i, in \$ per kg;

L_a = Land area used, in hectares;

L_c = Land area cost, in \$ per hectare per annum;

B = Net benefit to Manildra of being able to generate more biofuels, in \$ per annum;

T = Cost of financing, installing and maintaining tanks, in \$ per annum;

O = Operational cost (salaries, office, etc.), in \$ per annum;

d_q = Days in season q;

P_q = Seaweed production per hectare for season q, in dry weight kg per hectare;

where \$ are Australian dollars throughout, and T is derived using a standard amortising calculation over the expected life of the tanks:

$$T = \frac{T_c \times L_a \times L_p \times I}{1 - (1 + I)^{-N}} \tag{4}$$

where:

T_c = Cost of tank purchase, fitout and installation, in \$/hectare;

L_p = Proportion of land area that is covered by tanks (dimensionless, e.g. 0.6);

I = Interest rate with annualised compounding, as a decimal;

N = Expected life of tanks, in years.

The nutrient take-up rate

$$n_i^T = W_i \times G_q \times L_a \times \frac{d_q}{365} \tag{5}$$

where:

W_i = Dry-weight proportion of nutrient i in the seaweed;

G_q = Growth rate of seaweed in season q, in kg of dry weight per hectare per annum;

The nutrient available in the retentate

$$n_i^F = F_i \times d_q \tag{6}$$

where:

F_i = Mass of nutrient in retentate flow, in kg per day.

If the aim is to consume all nitrogen produced by Manildra, then the required land area

$$L_a = \frac{n_N^F \times 365}{G_{\text{winter}} \times W_N} \tag{7}$$

where:

G_{winter} = Growth rate of seaweed in the winter season (slowest growing), in kg of dry weight per hectare, per annum;

W_N = Dry-weight proportion of nitrogen in the seaweed.

Modelling features of the equations above include that:

- We assume that the following variables are constant throughout the year:
  - Flow rate of retentate from Manildra;
  - Market price of seaweed produced;

  - Take-up rates of nutrients per unit of dry weight seaweed produced;
  - Proportions of nutrients in the retentate;
  - Cost of adding nutrients to the water;
  - Cost of scrubbing nutrients from the water;
- Meanwhile, we assume that the following are seasonal:
  - Growth rate of seaweed;
  - Water evaporation.

The time scale is based on a 'season', which is simply a period over which a distinct growth rate is specified. Typically we would expect this to be quarterly, with growth rates specified over summer, winter, spring and autumn. However, we could also move to monthly seasonality (for example, if we wanted to use monthly-specified solar irradiation data). Depending on the deficit or accumulation of individual nutrients in the retentate flow, we will be required to either add nutrients to or remove nutrients from the water at a cost of n_i^A or n_i^R, respectively. We assume that there is a known cost for removal of any nutrient, for example by adding a downstream scrubbing tank. If we expect the nutrient to completely evaporate (for example a small amount of chlorine), then this removal cost may be considered to be zero. Water is treated in the same way as the nutrients, therefore if water needs to be added, then we simply need to include the cost as $n^A_{\mathrm{H_2O}}$. The land cost is represented by $[L_a \times L_c - B]^+$, therefore if it is demonstrated that the annual benefit to the Manildra biorefinery, B, exceeds the opportunity cost of the land (for example cattle adjustment), then we are counting on using the land at no cost. By assigning a price to each 'balancing' step (that is to maintain nutrient equilibrium in the tanks), the outcome may be readily reevaluated with adjustments to changing nutrient sources or mitigation techniques, or to comply with regulatory requirements. For instance, if we cannot release any zinc into the environment, then its mitigation cost is set prohibitively high.

Table 2: Results from the example models for one calendar quarter.

Variable	Scenario A		Scenario B	
	Summer	Winter	Summer	Winter
Land use (ha)	100	100	505	1264
Number of tanks	59,524	59,524	300,575	752,381
Upfront funding (\$m)	36.36	36.36	179.57	447.94
Total seasonal cost (\$m)	2.67	2.04	10.48	21.40
Seaweed produced (tonnes)	3,750	1,500	18,938	18,960
Seaweed revenue (\$m)	18.75	7.50	94.69	94.80
Gross margin (\$m)	16.08	5.46	84.20	73.40

3.2 Example Results

The model presented above was run for the two scenarios:

- Current output at Manildra (Scenario A); and
- Potential new retentate stream (Scenario B).

Table 2 presents the results for a single calendar quarter in either Summer or Winter. Spring and Autumn growth rates were not tested. In Scenario A, only 2.5 ha is required for consuming the retentate. However, for this model, land use is set to 100 ha with the assumption that Venus Shell Systems purchase additional nutrients. In contrast, for Scenario B, the land use is set to the land required to use all of the carbon and nitrogen in the retentate.

While the return on investment looks extremely favourable in these examples, the results are of course very much dependent on the model assumptions supplied to us by Venus Shell Systems. Table 3 lists these assumptions. For example, the assumption of a \$5/kg margin on every kg of seaweed sold (sale price minus drying, packaging etc.) will vary in real life according to the scale of the operation (as will the assumption that all seaweed produced is immediately sold).

Table 3: Assumptions for results in Table 2

Variable	Value
Dry weight seaweed gross margin ($/kg)	5.00
Funding rate (%pa)	12.00
Annualised seasonal growth per ha of water coverage (equivalent to 100% coverage) —Summer (dry weight tonnes)	200
Annualised seasonal growth per ha of water —Winter (dry weight tonnes)	80
Water depth (m)	0.60
Water coverage (% of land)	75
Water per hectare (m^3)	4500
Weekly water loss (%)	10
Efficiency of carbon uptake (%)	85
Area per tank (m^2)	12.60
Annual land opportunity cost ($)	50.00
Benefit to Manildra (waste disposal) ($)	100,000
Yearly operational cost ($m)	1.00

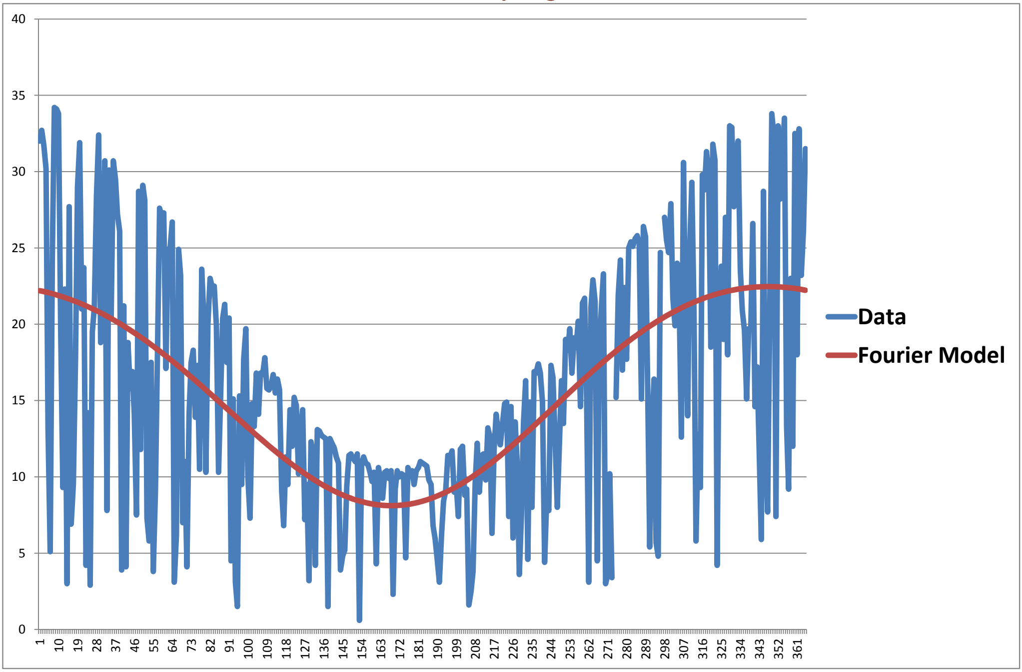
4 Solar Radiation Resource Assessment

The input solar energy is a significant contributor to the growth of the seaweed, so a potential concern is the effect of a sustained period of cloudy days. This could lead to a lack of sufficient energy input to maintain growth rates at the level that is required to continue throughput of the nutrients and carbon dioxide produced at Manildra. Here we conduct a preliminary assessment of the solar resource and give an indication how the issue may be approached.

Daily total solar radiation values are available from the Australian Bureau of Meteorology where there is also a precise definition of this measure.[1] These

[1] http://www.bom.gov.au/climate/data/

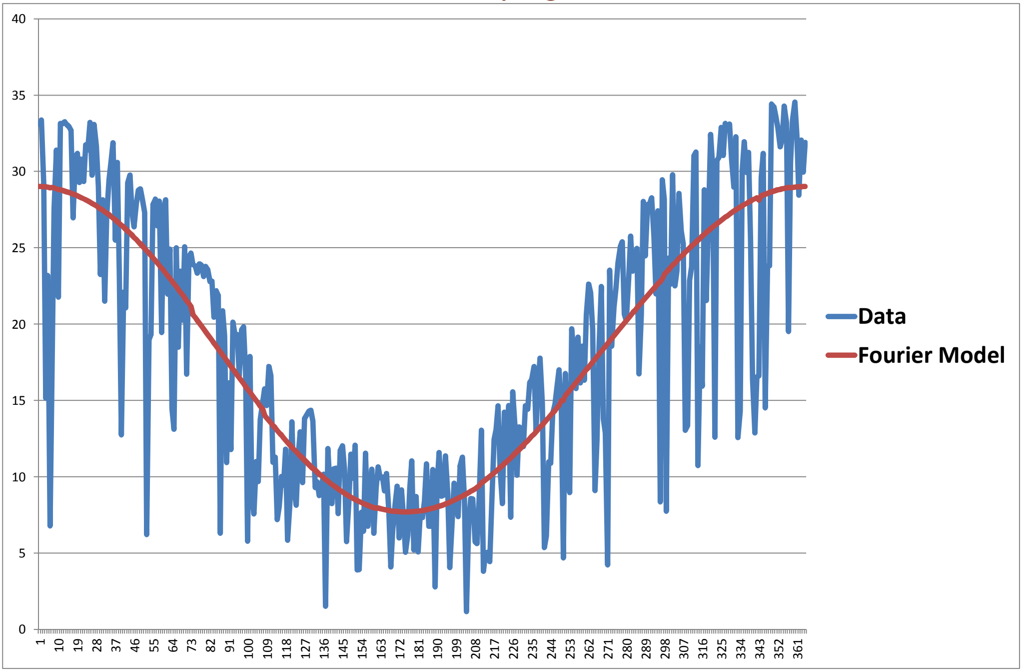
Figure 5: Milton daily total solar radiation over one year [MJ/m^2] plus best fit Fourier series model of the underlying values.



are for the years 1990 until the present (the beginning of 2014 for the MISG) and are estimated from satellite images.

The key question is: What is the frequency of cloudy days? Too many cloudy days in a row might impede the Venus Shell Systems processing system to the extent that waste from Manildra will have to be stored or worse dumped. To answer this question, we define what we will consider to be a cloudy day. To do that, we examine the total daily solar radiation for a Bureau of Meteorology site close to the project site, Milton NSW, and, for comparison, Adelaide SA, since the latter site is in a completely different climate zone, see Figures 5 and 6. These figures show the daily total solar radiation over one representative year (365 days). Superimposed on these graphs are curves

Figure 6: Adelaide daily total solar radiation over one year [MJ/m^2] plus best fit Fourier series model of the underlying values.



denoting the Fourier series model for all the data (1990–2013).

The differences between the two figures tell a lot about the climate of the two locations. Adelaide has a greater probability both of sunny days and also sequences of them. Milton exhibits more variable conditions day to day. We thus define a cloudy day at Milton as one where the total solar radiation is below the long-term average for that day. From this determination we proceed to estimate the probabilities of sequences of cloudy days, as given in Table 4. This information will aid the planners decide what level of use of the upstream Manildra outputs they can accept and at what level of risk. The probabilities given are for that exact number of cloudy days in succession with a non-cloudy day on either side.

Table 4: Estimated probabilities of sequences of cloudy days for Milton.

Number of cloudy days in a row	2	3	4	5	(exactly)
Probability	0.046	0.024	0.012	0.018	

5 Modelling seaweed growth over time

The following subsections model seaweed growth using ordinary differential equations considering production as a function of time. As indicated in the earlier sections, some modelling approaches deal with the bulk growth of seaweed by season or year; however, some effects take place over a smaller time scale. The further refinement of considering position within the tanks and using partial differential equations is not justified as the tanks are well-mixed. Several versions of the models were explored by the MISG team with the assistance of MATLAB routines. The models are still at a preliminary stage. Further work is needed to refine and possibly coalesce them and to find appropriate constants.

5.1 A simple water, carbon and nitrogen compartment model

In this subsection a model is constructed that considers the changes in time of water, carbon and nitrogen in the system. These are crucial quantities for seaweed production and consumption of retentate. A simple time-dependent mass balance approach is used with first approximation to the form of terms. This will permit

1. identification of term–by–term issues leading to more realistic terms,
2. comparison of steady state solutions with current operating data to determine model parameters,

3. future analysis of time–dependent features.

The equations relate to a single seaweed-production tank but readily scale to an array of tanks. The model has not been fully implemented.

Seaweed The basic compartment model assumes that the seaweed mass s [kg] is composed of portions relating to carbon c_s, nitrogen n_s, and water w_s.

$$s = c_s + n_s + w_s. \tag{8}$$

Some seaweed products are required to be nitrogen rich and the proportion n_s/s is a key qualitative measure for the seaweed.

Water Starting with a total tank with surface area $\text{SA}\,\mathrm{m}^2$ and volume $V = 0.6 \times \text{SA}\,\mathrm{m}^3$, we represent the mass of water $w(t)$ in the tank over time t [days] as

$$\frac{dw}{dt} = k_1\rho + (k_2 - k_3 - k_4)(\text{SA})\rho - k_5 w_s + k_6 \tag{9}$$

with parameters k_1 [m^3/day] representing the retentate inflow from the Manildra refinery, k_2 rainfall, and k_3 evaporation [m/day], as given by the Bureau of Meteorology for the region, whereas k_4 is a correction to the evaporation rate to take into account local variation and/or the impact of aeration, and k_5 is the harvesting rate [/day]. The constant ρ represents the density of water (retentate is assumed the same as fresh and salt water). The top–up rate, k_6, is determined by ensuring that the tank's water level is maintained and hence the time derivative (dw/dt) is zero. The rainfall and evaporation parameters are time dependent when necessary to account for seasonal or daily variations.

Carbon Simplistically, we can think of the growth of seaweed as being a conversion of carbon in the water, $c_w(t)$, into carbon in the seaweed, $c_s(t)$.

In this model the only source of carbon considered is CO_2 injected into the water $k_7(\cdot)$ [kg/day] where $(\cdot)$ denotes the unspecified dependency on such factors as the pH of the water.

$$\frac{dc_w}{dt} = k_7(\cdot) - k_8\,[G(\cdot)]\,c_s\,, \tag{10}$$

$$\frac{dc_s}{dt} = k_8\,[G(\cdot)]\,c_s - k_5 c_s\,. \tag{11}$$

The growth function $G(\cdot)$ with scaling parameter k_8 governs the growth. As an initial form, $G(\cdot)$ is chosen as

$$G = \frac{\lambda}{\varepsilon + s} n_w c_w\,. \tag{12}$$

This reflects the impact of nitrogen in the water n_w on driving the growth rate, and the requirement for carbon in the water for growth. The parameter λ represents the degree of solar irradiation and at this initial stage is considered constant. The component $1/(\varepsilon + s)$ captures the effect of shading by seaweed.

Nitrogen Simplistically, the growth of seaweed is proportional to the available nitrogen in the water n_w. However, nitrogen is also converted into stored nitrogen in the seaweed, and the amount stored per kilogram of seaweed varies with nitrogen supply. Notably, under conditions of starvation, when $n_w = 0$, approximately, stored nitrogen will be released and used to support growth. In the model, this is treated as a release of nitrogen into the water (n_w) and consumption by the seaweed from there. We introduce further parameters k_{10}, k_{11}, k_{12}, k_{13}, σ_1 and σ_2. The nitrogen in the water satisfies

$$\frac{dn_w}{dt} = k_{10}(\cdot) - k_{11}[G(\cdot)]c_s + k_{12}\frac{c_s n_s}{\sigma_1 + n_w} - k_{13}\frac{c_s n_w}{\sigma_2 + n_s}\,. \tag{13}$$

In order, the terms on the right-hand side of equation (13) represent nitrogen supply, nitrogen consumption for growth, release of stored nitrogen, and sequestration as stored nitrogen. The latter two terms have "Michaelis–Menten" form, so that the release of stored nitrogen, as well as being proportional to stored nitrogen in the seaweed, is limited by high levels of nitrogen in the water, and similarly the sequestration of stored nitrogen in the seaweed, although proportional to the nitrogen content of the water, is limited by high levels of nitrogen already stored in the seaweed. Meanwhile, the nitrogen in the seaweed satisfies

$$\frac{dn_s}{dt} = -k_{12}\frac{c_s n_s}{\sigma_1 + n_w} + k_{13}\frac{c_s n_w}{\sigma_2 + n_s} - k_5 n_s \,. \tag{14}$$

In order, these terms represent conversion to dissolved nitrogen, sequestering as stored nitrogen, and harvesting.

Within this model there is an underlying assumption that when carbon moves from the water to the seaweed, that the decrease in mass c_w equals the increase in mass of c_s. If the carbon masses are defined to include other elements in carbon compounds, or if other proportionate effects are included, then the parameter value k_8 in equation (11) is replaced by a scaled parameter k_9. To allow for similar effects for nitrogen, k_{12} and k_{13} in equation (14) are replaced with scaled parameters k_{14} and k_{15}.

5.2 Further models

Ren et al. [1] provides an ordinary differential equation for a seaweed ecosystem model. This and related work, such as the paper by Ren et al. [2], are a potential source of parameters for the models to complement those parameters known by Venus Shell Systems or available from the Bureau of Meteorology. One feature of the model is a separate treatment of nitrate and ammonium forms of the nitrogen in the water. A reduced form of the ecosystem model was explored, considering just seaweed population and the associated nitrogen using both nitrate and ammonium forms. Further to this model, other models were constructed using Michaelis–Menten kinetics for the carbon and nitrogen components. In contrast to subsection 5.1, this subsection uses concentrations rather than total masses. As with many investigations, it is not always clear

Table 5: Constants and parameters in the model of equations (15)–(19).

Notation	
X	mass of seaweed per total volume (V_t)
X_c	mass of carbon dioxide per volume of seaweed (V_s)
X_n	mass of nitrogen per volume of seaweed (V_s)
C	mass of carbon dioxide per volume of water (V_w)
N	mass of nitrogen per volume of water (V_w)
k_1	rate of uptake of carbon dioxide into the seaweed from the water
k_2	rate of uptake of nitrogen into the seaweed from the water
k_3	respiration rate (taking carbon dioxide in the seaweed out of the system)
k_4	photosynthesis rate
V_t	total volume of the tank ($V_w + V_s$)
ρ	density of seaweed (mass seaweed per volume of seaweed)
ε	half saturation constant
C_{total}	amount of non-carbon dioxide carbon in the seaweed

which unit is preferable and there are advantages and disadvantages with both approaches.

For the results presented here the effect of harvesting is not included. One group did explore the effect of different constant harvesting rates. However, in practice the harvesting rates will be adjusted to maintain an optimal amount of seaweed for growth. In the following we present one of the models which was used to illustrate the effect of different growth rates that are obtained as light varies during the passage of day and night. This effect is denoted sun factor. Table 5 gives other notation. To model the carbon in water and seaweed

$$V_w \frac{dC}{dt} = q_c(t) C_w^{in}(t) - \frac{k_1 X C}{1 + X_c/X_n} V_w \,, \tag{15}$$

$$V_s \frac{dX_c}{dt} = \frac{k_1 X C}{1 + X_c/X_n} V_s - k_3 X X_c V_s - k_4 X X_c X_n V_s \left(1 - \frac{V_s}{V_t}\right) \times \text{sun factor}, \tag{16}$$

and to likewise model the nitrogen in water and seaweed

$$V_w \tfrac{dN}{dt} = q_n(t) N_w^{in}(t) - \frac{k_2 X N}{1 + X_n/X_c} V_w \,, \tag{17}$$

$$V_s \tfrac{dX_n}{dt} = \frac{k_2 X N}{1 + X_n/X_c} V_s - k_4 X X_c X_n V_s \left(1 - \frac{V_s}{V_t}\right) \times \text{sun factor}. \tag{18}$$

In equations (15) and (18), the first term on the right-hand side is from the retentate influx. Finally the biovolume of the seaweed is

$$V_s \frac{dX}{dt} = k_4 X X_c X_n V_s \left(1 - \frac{V_s}{V_t}\right) \times \text{sun factor}. \tag{19}$$

Assuming that the carbon levels in the water are measured continuously and that the input is adjusted to maintain the carbon content at a constant level, C is assumed constant and equation (15) is removed from our model. Taking nitrogen levels to be similarly constant, which may be unrealistic as it is the flux inwards which we expect to be steady, and adding a half saturation constant and a control on the carbon dioxide concentration (to prevent it from reaching zero), the equations reduce to

$$\frac{dX_c}{dt} = \frac{k_1 X C}{\varepsilon + (X_c + C_{\text{total}})/X_n} - k_3 X X_c - k_4 X X_c X_n \left(1 - \frac{V_s}{V_t}\right) \times \text{sun factor}\,, \tag{20}$$

$$\frac{dX_n}{dt} = \frac{k_2 X N}{\varepsilon + X_n/(X_c + C_{\text{total}})} - k_4 X X_c X_n \left(1 - \frac{V_s}{V_t}\right) \times \text{sun factor}\,, \tag{21}$$

$$\frac{dX}{dt} = k_4 X X_c X_n \left(1 - \frac{V_s}{V_t}\right) \times \text{sun factor}\,. \tag{22}$$

Figures 7, 8 and 9 illustrate the growth of seaweed in the model for two forms of sun factor. The minimum sun factor has been chosen to be larger than zero. During photosynthesis (in daylight conditions) plants store a certain amount of energy as starch. During the night cycle, this starch is converted to sugar to continue the plant's growth in the absence of ambient light [3]. Geiger and Servaites [4] and Scialdone et al. [5] present more detailed modelling of this process. However, the present focus is to investigate the effect of changing sun levels. When averaged over the longer term (several days), using the more complicated, but maybe more realistic, tanh function (top frame of Figure 9) will not differ greatly to using the simpler sin function (Figures 7 and 8).

6 Conclusion

We considered the problem of growing seaweed using biorefinery retentate. Various models were constructed. A preliminary study of expected values of bulk inputs and outputs in the proposal appear well matched and suggest viability. In practice, there will be variability with time whether due to daily, seasonal or weather-driven fluctuations. The ordinary differential equation models presented here give an indication of how to consider these factors and possible effects.

Further opportunities are offered by the scope for growing a variety of strains of seaweed to produce a range of products for different market needs. Growth features and nutritional requirements vary with species and strain of seaweed. Varying production also improves the scope to more exactly match retentate production. Overall the project seems rather promising and exciting.

Acknowledgement We are grateful to Venus Shell Systems Pty. Ltd. and the industry representative Pia Winberg for bringing this problem to MISG 2014 and for her valuable input. We also acknowledge and thank other team members who worked on the problem: Scott Alexander, Joey Fung,

Figure 7: Sinusoidal form of sun factor (sunlight function) and a corresponding seaweed mass for a single day running from midnight to midnight.

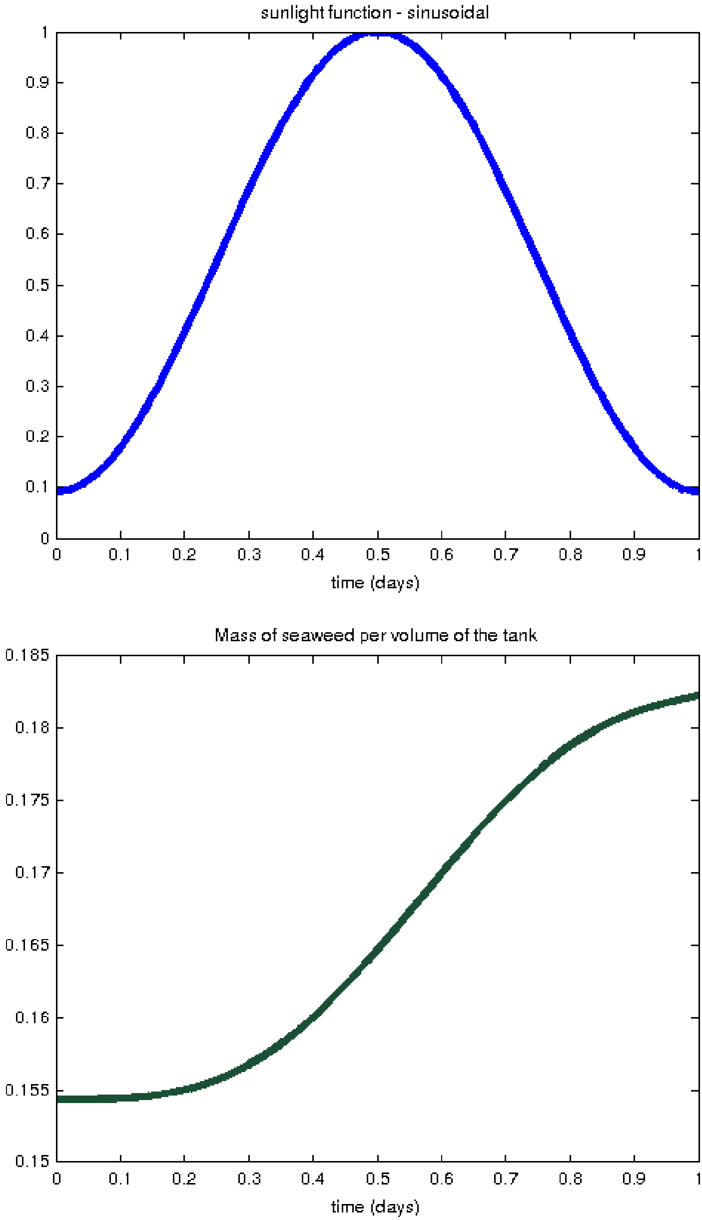


Figure 8: Sinusoidal form of sun factor (sunlight function) and a corresponding seaweed mass over one week. Days run from midnight to midnight.

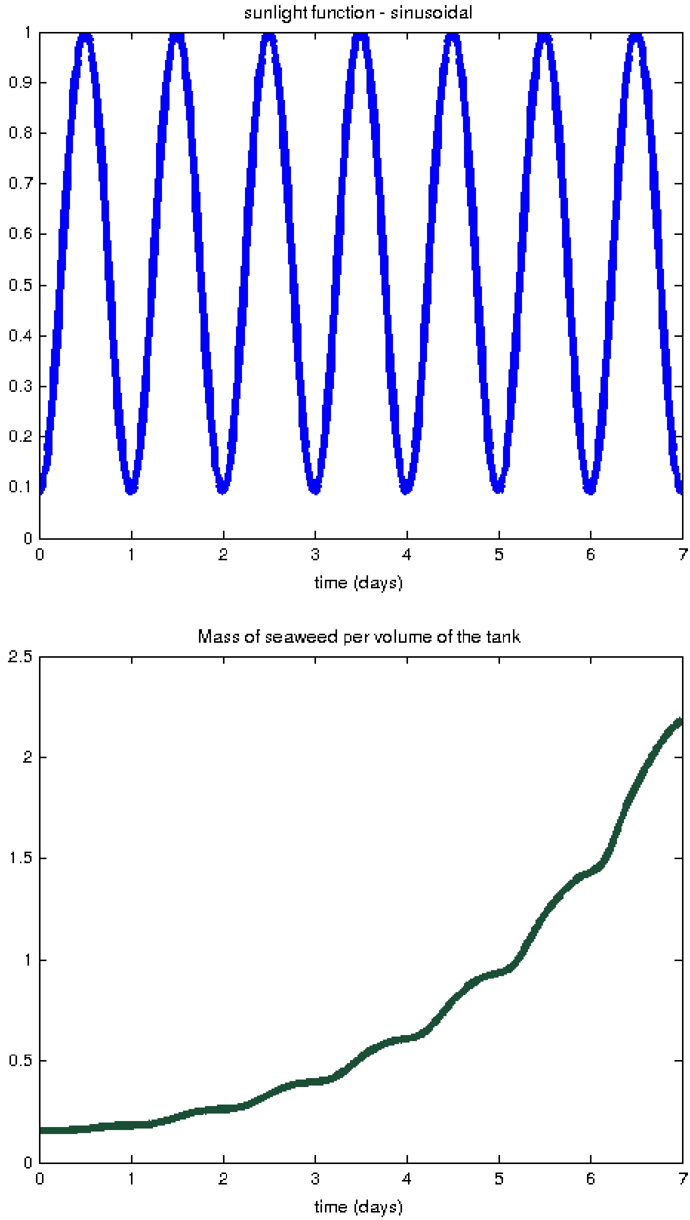
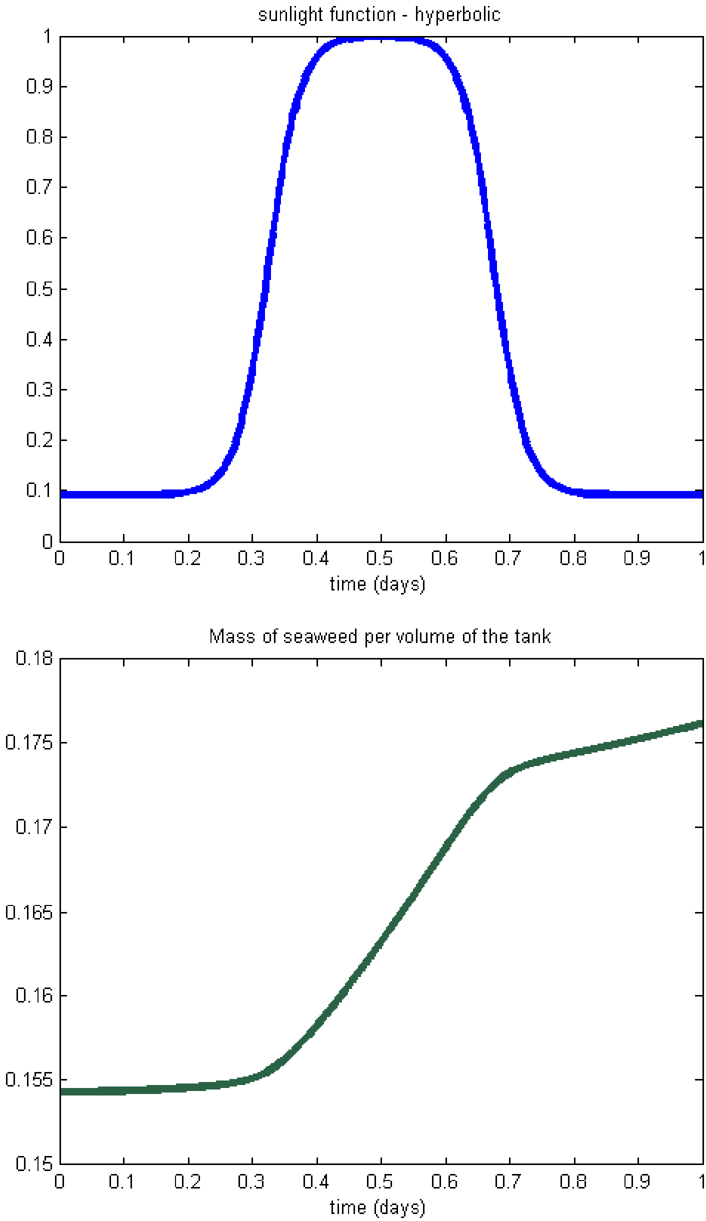


Figure 9: Hyperbolic tangent form of sun factor (sunlight function) and a corresponding seaweed mass for a single day running from midnight to midnight.



Alan Lee, Julio Losada, Dann Mallet, Bojana Manojlovic, Ali Murid and Norzieha Mustapha. The hospitality of our hosts at Queensland University of Technology was appreciated.

Acknowledgement of Geoff Mercer Geoff Mercer died a few weeks after the MISG. A supportive friend and colleague, he made valuable contributions to mathematics and to the mathematics community. His companionship is missed. He was always good to work with, projects together were enjoyable as well as productive. Fond memories remain of the time spent together.

References

[1] Jeffrey S. Ren, Jeanie Stenton-Dozey, David R. Plew, Jianguang Fang, Mark Gall (2012), "An ecosystem model for optimising production in integrated multitrophic aquaculture systems", *Ecological Modelling*, 246: 34–46. doi:10.1016/j.ecolmodel.2012.07.020 M21

[2] Jeffrey S. Ren, Alex H. Ross, Mark G. Hadfield, Barbara J. Hayden (2010), "An ecosystem model for estimating potential shellfish culture production in sheltered coastal waters", *Ecological Modelling*, 221: 527–539. doi:10.1016/j.ecolmodel.2009.11.003 M21

[3] Alison M. Smith, Mark Stitt (2007), "Coordination of carbon supply and plant growth", *Plant, Cell and Environment*, 30: 1126–1149. doi:10.1111/j.1365-3040.2007.01708.x M24

[4] Donald R. Geiger, Jerome C. Servaites (1994), "Diurnal regulation of photosynthetic carbon metabolism in C_3 plants", *Annu. Rev. Plant Physiol. Plant Mol. Biol.*, 45: 235–56. M24

[5] Antonio Scialdone, Sam Mugford, Doreen Feike, Alastair Skeffington, Philippa Borrill, Alexander Graf, Alison Smith, Martin Howard (2013),

"Arabidopsis plants perform arithmetic division to prevent starvation at night", *eLife* 2: e00669. doi:10.7554/elife.00669 M24

Author addresses

1. **Winston L. Sweatman**, Massey University, Auckland, New Zealand.
   mailto:W.Sweatman@massey.ac.nz
2. **Geoff Mercer**, NCEPH, Australian National University, Canberra, Australia.
3. **John Boland**, University of South Australia, Australia.
4. **Nicole Cusimano**, Queensland University of Technology, Australia.
5. **Ava Greenwood**, Queensland University of Technology, Australia.
6. **Kristen Harley**, Queensland University of Technology, Australia.
7. **Peter van Heijster**, Queensland University of Technology, Australia.
8. **Peter Kim**, University of Sydney, Australia.
9. **Joe Maisano**, Trading Technology Australia, and University of Technology, Sydney, Australia.
10. **Mark Nelson**, University of Wollongong, Australia.
11. **Graeme Pettet**, Queensland University of Technology, Australia.